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By

L. Lefton  
F. Whitlock

Goddard Space Flight Center  
Greenbelt, Maryland



## INTERPLANETARY TRAJECTORY ENCKE METHOD

### (ITEM) UPDATE 12C PROGRAM MANUAL

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This manual describes the content and modes of use of the Interplanetary Trajectory program by the Encke Method. It is a general purpose numerical integration program in interplanetary space permitting the inclusion of numerous perturbations. It is written in MAP language for the 7094 Mod II with hardware double precision. The program is based on a program originally conceived in 1959 by R. K. Squires, of the Goddard Space Flight Center and S. Pines and H. Wolf, then with Republic Aviation Corporation and was programmed under Contract NASW-109. Earlier versions have been described in Reference 7. This program has developed over a number of years under Contract NAS5-9085 and it is thus impossible to mention all those who have contributed. Major contributors to the program have been C. Bergen, C. Hipkins, L. Lefton, F. Shaffer and M. Wachman. The program has been widely distributed and is available for general use to interested organizations.\*\* A companion program coded in Fortran is also available.

---

\*Contractor with Analytical Mechanics Associates, Inc.

\*\*Address: Head, Special Projects Branch  
Laboratory for Theoretical Studies  
Goddard Space Flight Center  
Greenbelt, Maryland 20771

The present version is a further development of the program described in Ref. 7 on page i. In addition to the contributors already named on page i, F. H. Whitlock has contributed materially to the programming and the checkout of the additional modifications. The authors further wish to thank Mrs. A. Michalowski and Mrs. F. Magee for their assistance in the preparation of this manual.

It might be useful at this point to draw attention to some major changes effected in update 12C. They may be summarized as follows:

1. The Kepler problem has been completely reformulated in terms of universal variables and has been programmed in hardware double precision. This formulation is described in Appendix D.
2. The position and velocity vectors are carried in double precision, even though the perturbations are still computed in single precision only.
3. The perturbation acceleration computation has been changed to allow for the inclusion of additional planets as well as for additional harmonics of the earth and moon gravitational fields.
4. The planetary ephemerides are read from tapes derived either from cards supplied by the Naval Observatory or from the JPL ephemeris tapes. The coordinate system in each case is chosen as the mean equator and equinox of the middle of each two-year file.
5. The integration routine has been changed to permit integration in the negative time direction and the use of the universal anomaly as independent variable. The computation of the typical term  $R/r^3 - R_0/r_0^3$  in the planetary perturbations has been reformulated to avoid both the loss in accuracy and the binomial series expansion, as described in Appendix E.
6. A new trajectory search routine has been incorporated, using the techniques described in Reference 6. For this routine, the number of dependent and independent variables need not be equal. It is described in Section IV-H and Section VIII-C-1.
7. The integration and print interval control has been generalized and combined with the termination logic. It is described in Section VIII-G.

8. A new shadow routine has been added which is described in Section VIII-X and Appendix N.
9. The addresses on the modification cards are to be given in octal form, thus making laborious conversions unnecessary.
10. All cards read into the program (modifications as well as regular input) are printed, thus a record of all the parameters introduced into a run is kept.

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## I. INTRODUCTION

This report describes a general purpose Interplanetary Trajectory Encke Method (ITEM) Program, programmed for the IBM 7090 and IBM 7094. The method employed is designed to give the maximum available accuracy without incurring prohibitive penalties in machine time. On the basis of research described in Reference 4, the Encke method was selected as best satisfying these requirements. However, the classical Encke method was modified to eliminate some of its objectionable features. This modified Encke method is described in Appendix A.

The perturbations included in this program are the gravitational attractions of earth, moon, sun, Jupiter, Venus and Mars considered as point masses. Additionally, the effects of the second, third and fourth zonal harmonics and the tesseral harmonics of the earth and moon gravitational fields, as well as the aerodynamic drag, small corrective thrusts and radiation pressure including the shadow effect of the earth, are considered. The input may be prepared in any one of several common systems and a great variety of output options is available.

## II. NOTATION

Upper case - vectors; Hats - unit vectors; Lower case - magnitudes

<u>Description</u>	<u>Symbol</u>	<u>Units</u>
Cartesian coordinates of vehicle with respect to reference body	$x \ y \ z$	km
Velocity components in Cartesian coordinates	$\dot{x} \ \dot{y} \ \dot{z}$	km/sec
Time	$t$	hrs.
Longitude measured from Greenwich, + East (used in Section IV and Appendix H)	$\theta$	degrees
Longitude of vernal equinox	$\theta_0$	degrees
Speed	$v$	km/sec
Geodetic altitude*	$h$	km
Geodetic latitude	$\phi$	degrees
Geodetic flight path angle	$\gamma$	degrees
Geodetic flight path azimuth	$A$	degrees
Acceleration parameter (defined in Appendix E)	$u$	
Right ascension	RA	degrees
Astronomical units	AU	
Earth radii	ER	
Earth mass	$m_e$	

---

\*Note: The following 3 symbols with primes denote the corresponding geocentric quantities.  
Geocentric in this report refers to a spherical earth, i. e.,  $e^2 = 0$ . In this case  
 $\phi' = \delta$  = declination.

<u>Description</u>	<u>Symbol</u>
Vehicle position vector	R
Distance to vehicle	r
Perturbation displacement vector	$\Delta R$
Perturbation displacement vector components	$\xi, \eta, \zeta$
Perturbation acceleration	F
Coordinate functions and their time derivatives respectively	$f, g, \dot{f}, \dot{g}$
Mass parameter	$\mu$
Semi-major axis	a
Earth's eccentricity as used in Appendices H, I, L, S	e
Mean motion	n
Unit vectors for classical two-body orbit solution	$\hat{P}, \hat{Q}$
Eccentricity anomaly as used in Appendix D	E
Elevation angle as used in Appendix I	E
$R_0 \cdot \dot{R}_0$	$d_0$
Incremental eccentric anomaly as used in Appendix D	$\theta = E - E_0$
Functions of $\theta$ defined by equations (E.2)	$f_0, f_1, f_2, f_3, f_4$
Inclination of orbital plane	i
Right ascension of the ascending node	$\Omega$

<u>Description</u>	<u>Symbol</u>
Argument of perigee	$\omega$
Parameters which account for polar oblateness of the earth, defined in Appendix H	$c, s$
Right ascension of the station meridian	$RA_s$
Range measured from observation station	$\rho$
Direction cosines measured in a topocentric coordinate system	$\lambda, \mu, \nu$
Declination	$\delta$

#### SUBSCRIPTS

Vehicle	$v$
ith perturbing body	$i$
Quantity obtained from Keplerian solution of two-body problem	$k$
Reference body as used in Appendix B	$c$
Station	$s$
$R_A - R_B$	$R_{AB}$
Value at rectification time	$o$
Corresponds to x, y, z components respectively	$n = 1, 2, 3$
Value at perigee	$p$

### III. GENERAL PROCEDURE FOR USING PROGRAMS

Initial conditions, terminal conditions and print frequency, as well as other parameters controlling the flow of the program, are read as input. The computation of the trajectory then proceeds until one of the terminal conditions (e.g., maximum time) has been reached or an error is encountered. At this time the program prints the reason for its termination and selected storage locations, then proceeds to the next case. When an end of file is encountered on the input tape, control is transferred to the monitor.

A modification card which causes a core dump to be printed is available. An option is also included which permits the program to iterate on initial conditions for trajectories in order to satisfy a set of desired conditions at the moon, earth, or a designated planet.

#### IV. INITIAL CONDITIONS

The initial conditions necessary for the specification of a trajectory are:

1. Initial position of the vehicle relative to the reference body.
2. Initial velocity of the vehicle relative to the reference body.
3. Initial time of launch referenced to a base time.

For specification of the initial conditions, the reference systems and units shown below may be used.

##### A. Cartesian Coordinates

The coordinate system is defined as follows:

1. The origin is at the center of the reference body.
2. The x-axis is in the direction of the mean equinox of December 31.0 the year specified as year of launch.
3. The xy-plane is the mean equatorial plane of the earth.  
Initial position is given by the x, y, z coordinates of the vehicle.  
Initial velocity is given by the  $\dot{x}$ ,  $\dot{y}$ ,  $\dot{z}$  components of the vehicle.  
Initial time of launch from base time<sup>1</sup> (t) is also given. If the program is used in its standard form, the units<sup>2</sup> to be used for the above are:

x, y, z - km

$\dot{x}$ ,  $\dot{y}$ ,  $\dot{z}$  - km/sec

t - days,<sup>3</sup> hours, minutes, seconds from base time.

The year of launch must also be given.

---

1 The base time is 0.0h U.T. December 31 of the year previous to the year of launch.

2 Scale factors are used to convert the input units to the units used internally (ER or AU and hrs). Any other set of units may be used by changing these scale factors with a modification card as described in Section VIII.

3 The number of days must be an integer. Hours are expressed as floating point decimals; minutes and seconds may be included in the number of hours as a fractional part. Zeros must then be loaded in place of minutes and seconds.

B. Geodetic Polar Coordinates

Initial position of the vehicle is given by:

1. Geodetic latitude ( $\varphi$ )
2. Longitude, <sup>4</sup> measured from Greenwich ( $\theta$ )
3. Geodetic altitude (h)
4. Longitude of <sup>4</sup> vernal equinox at initial time ( $\theta_0$ ). This quantity may be computed by the program or may be loaded. (See IX-A Note 15)

Initial velocity of the vehicle is given by:

1. Speed (v) with respect to the center of the earth.
2. Flight path azimuth (A) measured clockwise from north in a plane normal to the geodetic altitude.
3. Flight path angle ( $\gamma$ ) measured from a plane normal to the geodetic altitude.

Initial time of launch from base time (t) must also be given.<sup>1</sup>

The following units must be used with the above:

1.  $\varphi$ ,  $\theta$ , and  $\theta_0$  - degrees; h - km.
2. A and  $\gamma$  - degrees; v - km/sec.
3. t - days, hours, minutes and seconds.

C. Geocentric Polar Coordinates

Ordinarily an input given in polar coordinates will be interpreted as described in paragraph B (preceding). However, if the eccentricity of the earth is set to zero, the program will interpret latitude as declination,

---

4 If the right ascension (RA) at initial time is known, it may be used in place of longitude ( $\theta$ ). The longitude of the vernal equinox ( $\theta_0$ ) is then set to zero.

height as distance above a spherical earth of equatorial radius, and flight path angle and azimuth with reference to a plane normal to the radius vector. If this interpretation of the input is desired, the eccentricity of the earth ( $e^2$ ) is set to zero with a modification card described in Section VIII-A. This eccentricity is used only in this part of the program. Therefore, other portions of the program will not be disturbed. An option is also included which interprets latitude and height as geodetic, and azimuth and flight path angle as geocentric. (See VIII-B-1)

#### D. Polar Coordinates in Moon Reference

If polar coordinate input is used in moon reference, the input quantities are interpreted in a righthanded coordinate system defined as follows:

1. The  $xy$ -plane is the moon's orbital plane.
2. The  $x$ -axis points toward the earth.
3. The  $z$ -axis is parallel to the angular momentum vector of the moon about the earth. In this system,

latitude  $\varphi$  is

the angle between radius vector and  $xy$ -plane,  
positive for positive  $z$ .

longitude  $\theta$  is

the angle between the projection of the radius vector  
on the  $xy$ -plane and the  $x$ -axis, positive for  
positive  $y$ .

$\theta_0$  irrelevant

altitude  $h$  is

the distance from the surface of a spherical moon.

speed  $v$  is

the speed with respect to the center of the moon in  
a non-rotating coordinate system.

Flight path azimuth and angle are defined in a plane normal to the radius vector.

## E. Translunar Plane

The translunar plane input (see Figure 1) uses the initial conditions (height,  $h$ , speed,  $v$ , argument of radius,  $\Psi$ ,<sup>5</sup> and flight path angle,  $\gamma$ ) with respect to the translunar plane. The inclination  $i_{TL}$  and the lead angle  $\phi$ <sup>6</sup> serve to position the translunar plane with respect to the moon's orbital plane. The moon's orbital plane is related to the standard coordinate system by means of the ephemeris as described in Appendix W.

The translunar plane (see Figure 1) input consists of the following:

Load - 1 = Lead angle ( $\phi$ ) - degrees

Load = Case No.

Load + 1 = Argument of radius ( $\Psi$ ) - degrees

Load + 2 = Altitude - km

Load + 3 = Speed - km/sec

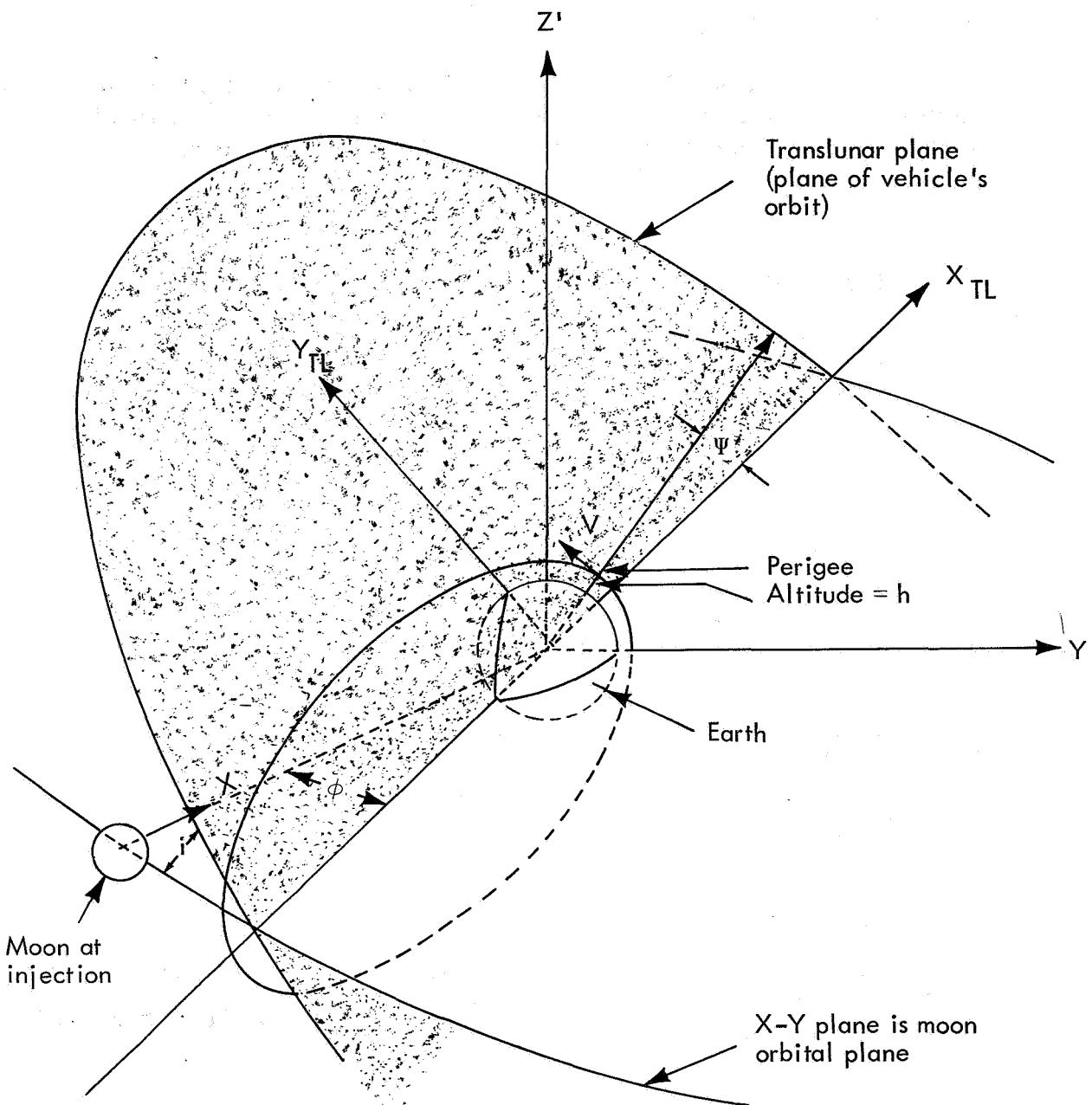
Load + 4 = Inclination of plane - degrees

Load + 5 = Flight path angle - degrees

The remainder of the load is the normal polar load.

---

- 5 The angle between the initial position vector and the ascending node with respect to the moon's orbital plane.
- 6 The lead angle is the angle between the position vector of the moon at launch and the descending node of the translunar plane with respect to the moon's orbital plane.



Program symbol:

- $\phi$  = Lunar lead angle
- $i$  = Inclination of translunar plane
- $\psi$  = In-plane angle of radius = argument of radius
- $\gamma$  = Flight path angle, angle above local horizontal (geo or lunisolar)
- $h$  = Injection altitude

Figure 1. Translunar Plane

#### F. Ecliptic Coordinates

Input in ecliptic coordinates is available. The coordinate system used in this case is identical to the coordinate system described in Section IV-A, with the ecliptic plane replacing the equatorial plane as x y-plane.

#### G. Osculating Element Input

One may input osculating elements as follows:

LOAD 1 = 4

LOAD -1 = Indicator for type of input

1 = Time of perigee

2 = Delta - M

3 = E

4 = True anomaly

LOAD = Case No.

LOAD +1 = Argument of perigee

LOAD +2 = Longitude of ascending node

LOAD +3 = Inclination

LOAD +4 = Semi-major axis (in earth radii)

LOAD +5 = Eccentricity

LOAD +6 = Time of perigee, mean anomaly, eccentric anomaly, or true anomaly (depends on whether LOAD -1 has a 1, 2, 3 or 4, respectively).

The program converts the above to Cartesian coordinates and then continues normally.

## H. Trajectory Search

The trajectory search can determine the initial conditions (in polar form) necessary to achieve any set of constraints chosen from among impact parameters and osculating elements at a target or a return reference body. Its activation is described in VIII-C-1.

The partial derivatives are computed by the secant method, thus, in general, a large number of trajectories must be computed. In the initial stages of the search it is recommended that the patched conic mode of computation be used. Further, throughout the iteration, the generalized anomaly should be used as independent variable. The method used in this search is described in Reference 6.

## I. Comments

1. The program computes in Cartesian coordinates.

The units used internally in the computation are:

- a. position: Earth radii (ER) - Astronomical units (AU)
- b. velocity: ER/hr - AU/hr
- c. time: hours

(Earth radii units are used in the earth or moon reference.  
Astronomical units are used in the sun or planet reference.)

2. Cartesian or polar coordinates can be used when launching from any body, providing ecliptic polar coordinates are used in sun or planet reference.
3. Equations for converting the initial conditions from polar to Cartesian coordinates are shown in Appendix H.

## V. TERMINATING CONDITIONS

The set of conditions which will terminate a trajectory may be summarized as:

1. Maximum time of flight - hrs.
2. Maximum distance from any possible reference body considered in the solution.
3. Minimum distance from any possible reference body considered in the solution.

Any of these conditions will terminate a trajectory. Loading a large number into any of the maxima and a zero into any of the minima will make the corresponding conditions inoperative. A proper choice of these numbers will permit complete computation of the desired trajectory, avoid extensive unnecessary computation and guard against faulty input.

## V. MOON'S ORBITAL PLANE INPUT AND OUTPUT

A polar coordinate system is available for input and output which uses as its reference plane the moon's orbital plane and the vector from moon to earth as unit vector. Polar coordinates in this system are defined analogous to geocentric polar coordinates. The cartesian coordinates in this system are computed by equations (H.3) with

$$c = s = r_B$$

and

$$\theta_0 = 0$$

Here  $r_B$  is the radius of the body of departure (earth or moon).

These coordinates are then transformed to equatorial coordinates by a matrix C computed as follows:

$$\hat{i} = \frac{\mathbf{R}_{EM}}{r_{EM}}$$

$$\hat{k} = \frac{\mathbf{R}_{EM} \times \dot{\mathbf{R}}_{EM}}{|\mathbf{R}_{EM} \times \dot{\mathbf{R}}_{EM}|} \quad (V.1)$$

$$\hat{j} = \hat{k} \times \hat{i}$$

The transformation matrix  $C$  is then given by

$$C = \begin{pmatrix} i_x & j_x & k_x \\ i_y & j_y & k_y \\ i_z & j_z & k_z \end{pmatrix} \quad (V.2)$$

and

$$R = CR_{MOP} \quad (V.3)$$

$$\dot{R} = CR_{MOP}$$

The matrix  $C$  is unitary, and  $C^{-1} = C^*$ , permitting easy inversion of equations (V.2).

## VI. PERMISSIBLE PERTURBATIONS

The trajectory computation consists of two parts, the exact solution to the two-body problem and integrated additions to this solution for the effect of perturbations. The successful control of round-off errors in the modified Encke method depends on preventing the accumulated round-off error in the integrated perturbation displacement from affecting the computed position. This is achieved by always keeping the perturbation displacement small by rectifying whenever the perturbation exceeds specified limits. The constants mentioned below are used in determining the allowable limits as ratios of the perturbation position and velocity to the two-body position and velocity, respectively.

This ratio is shown for the position vector in Figure 2.

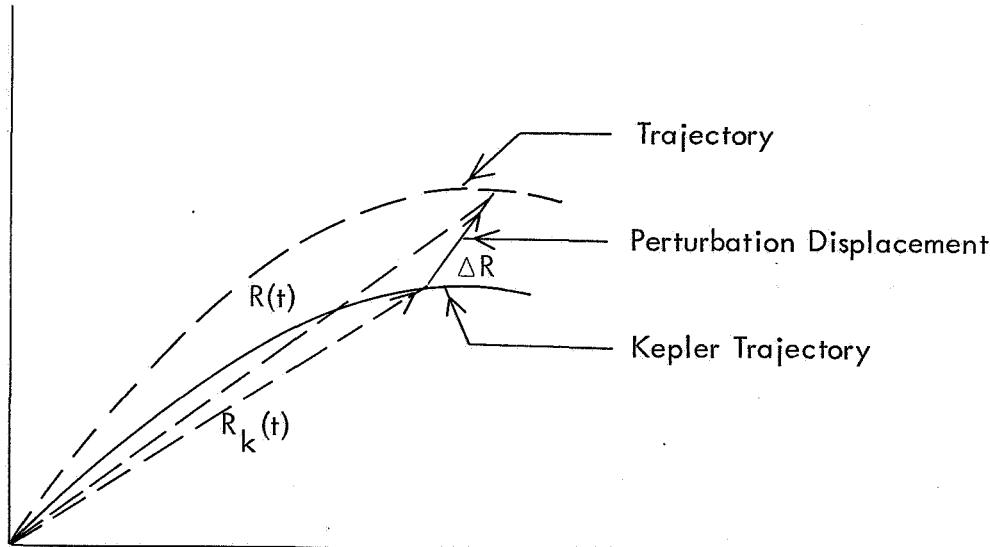


Figure 2. Encke Method

The recommended values for these ratios are as follows:

$$\frac{\Delta r}{r} \leq .01$$

$$\frac{\Delta \dot{r}}{\dot{r}} \leq .01$$

$$u^* \leq .01$$

---

\*The parameter  $u$  is defined in Appendix E.

## VII. RADAR INFORMATION PROGRAM

The program may be used to simulate radar data if desired. A maximum of 30 stations can be handled at one time. The following information is required for each station considered:

1. Station name - for identification purposes
2. Position of radar station
  - a. Longitude ( $\theta$ ) of the station from Greenwich - degrees, minutes, seconds - positive eastward\*
  - b. Geodetic latitude ( $\varphi$ ) of the station - degrees, minutes, seconds - positive north\*
  - c. Altitude (h) of station above sea level - feet

The simulated radar information consists of azimuth, elevation, topocentric right ascension and declination, slant range and range rate. It is printed at every normal print time for which the elevation angle is positive. Refraction is not considered.

This part is coded as a subroutine and may be called at other times, if desired.

In addition, the radar program will now print topocentric right ascension and declination as well as Minitrack direction cosines. A mode is available which differences the range and Minitrack direction cosines for two consecutive runs and prints a summary of these differences. The effect of various parameters on these measurements can thus be conveniently studied.

---

\* Alternatively, these quantities may be given in degrees and decimals. 0's must be loaded into the positions reserved for minutes and seconds.

The fractional parts will not appear in the printout reproducing the station coordinates. They will, however, be included in the computation.

## VIII. MODIFICATION CARDS

Normally, the program proceeds in the fashion described in Section IX. Modification cards may be used to operate the program in non-standard fashion to provide for special requirements. Any modification card included in a case will be operative for all succeeding cases in the input stack, unless it is revoked. Modification cards are listed in this report with their octal and symbolic locations. The octal locations refer to the "update 12C" version of the program, and some may be different for later update versions. If such a later version is used, the octal locations and the addresses should be checked in the symbol table at the end of the assembly listing. The printout contains the update number used, following the title print. The octal location is to be punched in columns 2-6, columns 8-10 contain one of the following operation codes

DEC, OCT, or BCD

A number to be loaded must begin in column 12 for DEC<sup>1</sup> and OCT cards. The first blank encountered to the right of column 12 terminates reading. A BCD card must have a word count<sup>2</sup> in column 12, blanks will not terminate reading of BCD cards.

Some modification cards which are frequently used are described below:

### A. Geocentric Polar Coordinates (i.e., spherical earth)

Modification card

30111 IECC                    DEC 0\*

---

- 1 A DEC format will read in fixed or floating point numbers. A floating point conversion is performed if the number includes a decimal point or the symbol E.
- 2 The number of words is the number of symbols (including blanks) divided by 6. A word count of 10 is assumed if column 12 is blank.

\* Fixed point numbers

To revoke modification card:

30111 IECC DEC .006693422\*\*

## B. Cartesian Scale Factor - Input and Output

30112 REKM	DEC	....	(1)
30025 MDKM	DEC	....	(2)
30026 TSCL	DEC	....	(3)

where

Example: Initial conditions to be in ER and ER/hr. Then card reads:

30112 REKM	DEC 1.	** (1)
30025 MDKM	DEC 23454.87, 1.	** (2), (3), (4)

To revoke modification card:

30112 REKM	DEC	6378.165	**
30025 MDKM	DEC	14.9599 E 7, 3600.	**

(4) Several numbers appearing on one card, separated by commas, are loaded into consecutive locations.

### C. To Initiate Radar

### Modification cards (2 cards)

12654 RADSTA OCT 007400423630 TSX Z\$BGN1, 4

The above card activates the loading of station coordinates

## \*\* Floating point numbers

14771 RADENT OCT 007400424002 TSX ZRADAR, 4

The above card activates the computation of radar information. To revoke modification cards (2 cards):

12654 RADSTA OCT 076100000000 NOP (1)

The above card inactivates the loading

(1) This card must be included with the first repeat case if identical station locations are used.

14771 RADENT OCT 076100000000 NOP

The above card inactivates the computation.

D. Table Print - Prints astronomical tables in core at every rectification.

Modification card:

13447 TBPRNT OCT 007400425153 TSX TAPRN, 4

To revoke modification card:

13447 TBPRNT OCT 076100000000 NOP

E. Table Print Exits

Normally the program continues after Table Print.

Modification cards:

If desired to stop:

25161 TARTN DEC 0 HTR

To do next case:

25161 TARTN OCT 002000020310 TRA SWT 5

To revoke modification card:

25161 TARTN OCT 002000400001 TRA 1, 4

**F. End Card**

Normally the program returns control to the Monitor after all the cases are completed. If a program stop is desired, the following card is added after the input.

37776 SWEF OCT 076100000000 NOP

G. Integration and Print Intervals are controlled by distances from reference body. Seven different intervals are possible.

In earth reference the following is normal:

30176 ERVEC	DEC 1., 4., 125., 0, 0, 0, 0, 0	Radius vectors**
30206 EIVEC	DEC .03125, .5, 0, 0, 0, 0, 0, 0	Integration intervals**
30216 EPVEC	DEC 1., 6., 0, 0, 0, 0, 0, 0	Print intervals**

Between 1 and 4 earth radii, the integration interval will be .03125 hours and the print interval will be 1. hour. Between 4 and 125 earth radii, the integration interval will be .5 of an hour and the print interval will be 6 hours.

In moon reference the following is normal:

30226	MRVEC	DEC	.27, 10., 0, 0, 0, 0, 0, 0	**
30236	MIVEC	DEC	.0625, 0, 0, 0, 0, 0, 0, 0	**
30246	MPVEC	DEC	1., 0, 0, 0, 0, 0, 0, 0	**

In sun reference the following is normal:

30256	SRVEC	DEC	.1, 10., 0, 0, 0, 0, 0, 0, 0	**
30266	SIVVEC	DEC	24., 0, 0, 0, 0, 0, 0, 0	**
30276	SPVEC	DEC	48., 0, 0, 0, 0, 0, 0, 0	**

In a planetary reference the following vectors should be read in:

30306 PRVEC	DEC (2 to 8 values)	**
30316 PIVEC	DEC (1 to 7 values)	**
30326 PPVEC	DEC (1 to 7 values)	**

## \*\* Floating point numbers

ERVEC and MRVEC should be in earth radii units. SRVEC and PRVEC should be in astronomical units. EIVEC, EPVEC, MIVEC, MPVEC, SIVEC, SPVEC, PIVEC and PPVEC should be in hours or fractions of an hour.

H. Inclusion or Exclusion of Perturbations

1. Ordinarily included are the gravitational field of the earth (2nd, 3rd and 4th zonal harmonics) and the gravitational attractions of sun, moon, Mars, Venus and Jupiter. To exclude any or all of these perturbations, put 0 into the corresponding locations as follows:

30067 MS1	DEC ....	Sun**
30066 MM1	DEC ....	Moon**
30072 MJU1	DEC ....	Jupiter**
30071 MMR1	DEC ....	Mars**
30070 MVE1	DEC ....	Venus**
30101 J20	DEC ....	coefficient of 2nd zonal harmonic**
30102 J30	DEC ....	coefficient of 3rd zonal harmonic**
30103 J40	DEC ....	coefficient of 4th zonal harmonic**

2. The perturbation due to the moon's oblateness is computed in moon reference. If it is to be omitted, put 0 into

30142 CONSC      DEC 0      \*

3. The following numbers are to be loaded to restore standard operation of the program:

30067 MS1	DEC 332951.3	**
30066 MM1	DEC .01229483	**
30072 MJU1	DEC 317.887	**
30071 MMR1	DEC .1078210	**
30070 MVE1	DEC .8147689	**
30101 J20	DEC .0010823	**

\* Fixed point numbers

\*\* Floating point numbers

30102 J30	DEC -2.3 E -6	**
30103 J40	DEC -1.8 E -6	**
30142 CONSC	DEC .36366998 E -2	**

The following perturbations are not ordinarily included:

4. Radiation Pressure may be included by loading a coefficient into

30105 RACOE	DEC ....	**
-------------	----------	----

The number to be loaded is:

$$\frac{KC_r A}{m}$$

$C_r$  is the radiation pressure in dynes/cm<sup>2</sup> at a distance of 1 AU from the sun.

$$(C_r = 4.6 \times 10^{-5} \frac{\text{dynes}}{\text{cm}^2}) \quad (\text{Estimated value})$$

A area in cm<sup>2</sup>

m mass in grams

$$K \text{ scalar } 3600^2 (23455.)^2 / 6378.165 \times 10^5 = .11178 \times 10^8$$

sec to hrs, ER to AU, cm to ER

The radiation pressure will only be active if sunlight impinges on the vehicle. For correct results, the radiation pressure should therefore be run only in conjunction with the optional shadow computation as described in Appendix O.

If, however, the expected trajectory may be safely assumed to be entirely out of the earth's shadow, shadow testing may be avoided, with a consequent saving in machine time. In this case, the following modification card must also be included:

33047 SHADIN	DEC 1.	**
--------------	--------	----

5. If Inclusion of the Aerodynamic Drag is Desired, the drag parameter  $1/2 C_D A/m$  must be loaded in location COEFL by means of the following card:

---

\*\* Floating point numbers

30104 COEFL            DEC ....            \*\*

The units of  $C_D$  A/m are the area in  $\text{cm}^2$  and the mass in grams.

A layered atmosphere rotating with the earth is assumed. The density is obtained by a linear interpolation of density-altitude tables.

6. To include tesseral harmonics, load

30153 TESSTR            DEC 1            \*

Normal tesseral coefficients are as follows:

30114 J22	DEC -1.9 E -6	$J_{22}^{**}$
30115 L22	DEC -21.	$\lambda_{22}^{**}$
30116 J31	DEC -1.51 E -6	$J_{31}^{**}$
30117 L31	DEC 0.	$\lambda_{31}^{**}$
30120 J33	DEC -.149 E -6	$J_{33}^{**}$
30121 L33	DEC 22.8	$\lambda_{33}^{**}$

Moon oblateness coefficients are normally:

30106 JM20	DEC 15.378959 E -6	$J_{20}^{**}$
30107 JM30	DEC 0.	$J_{30}^{**}$
30110 JM40	DEC 0.	$J_{40}^{**}$
30127 JM31	DEC 0.	$J_{31}^{**}$
30130 LM31	DEC 0.	$\lambda_{31}^{**}$
30131 JM33	DEC 0.	$J_{33}^{**}$
30132 LM33	DEC 0.	$\lambda_{33}^{**}$
30125 JM22	DEC 0.	$J_{22}^{**}$
30126 LM22	DEC 0.	$\lambda_{22}^{**}$

---

\* Fixed point numbers

\*\* Floating point numbers

7. If any of these perturbations are to be omitted in repeat cases, zero must be loaded in the corresponding locations.

In order to provide a permanent record of the perturbations included in each particular trajectory, the numbers appearing in the locations described in Section VIII-H are printed at the beginning of each trajectory.

I. Atmospheric Tables for the Drag Computation are stored in core. They are contained in the COSPAR Report accepted for the April 1961 Meeting in Florence, Italy, "CIRA 1961." If it is desired to change this atmosphere, the following procedure has to be followed:  
Modification cards

33747 NTAR	DEC ....	the number to be entered is N-1, where N is the number of entries in the density table.*
------------	----------	--

33750 RTBL	DEC ..., ..., ...,	the values of r at which the density is given in ascending order, a maximum of 54.**
------------	--------------------	--

The units for the air density are grams/cm<sup>3</sup> and the height is given in ER from the center of the earth.

34037 RHOT	DEC ..., ...,	the values of the air density in gm/cm <sup>3</sup> in respective order corresponding to the r table.**
------------	---------------	---

If other units are used for the density table, the drag parameter described in Part H of this section has to be read in the same units and the constant (-6378.165 E 5)\*\* normally in

27763 DRSC	DEC ....	has to be changed accordingly.**
------------	----------	----------------------------------

The negative sign directs the drag force opposite to the velocity. This constant converts the drag from the units used for A, m, and  $\rho$  to ER/hr<sup>2</sup>.

---

\* Fixed point numbers

\*\* Floating point numbers

J. Angle from Ascending Node to Vehicle

To print angle add the following card:

26017 EAFANS OCT 002000034701 TRA ASCSAT

To revoke:

26017 EAFANS OCT 076100000000 NOP

K. The Program Provides a Special Printout described in the output (Section X) near the earth, moon, sun and planets. This printout occurs at every integration step and is useful to observe the behavior of these relevant quantities during ascent and re-entry. This feature is triggered by the following modification cards:

30336 SERE	DEC ....	earth reference (ER)**
30337 SMRE	DEC ....	moon reference (ER)**
30340 SSRE	DEC ....	sun reference (AU)**
30341 STRE	DEC ....	planet reference (AU)**

The numbers used above are the radial distance within which the special print is to be effective. The units are Earth Radii for the earth and moon references and Astronomical Units for sun and planet references. A zero in any of the above locations suppresses this feature.

L. The Rectification Print may be eliminated with the following cards:

13607 RCTPR+4	OCT 076100000000, 076100000000	(1)
13627 RECTPS	OCT 076100000000, 076100000000	(2)

In order to obtain the rectification prints in repeat cases, after they have been eliminated in one case, the original contents, as given in the assembly listing of these locations have to be restored.

- (1) Eliminates ratio print
- (2) Eliminates normal rectification print

---

\*\* Floating point numbers

In order to get the normal print at  $t = 0$  (which is also a rectification), the program tests the time. If zero, it prints normal output, otherwise the skip print routine is effective.

M. To Compute and Print Latitude, Longitude and Velocity at Perigee  
add the following card:

25741 PGCOOV OCT 076100000000 NOP

### N. Print at Ascending Node, Apogee and Perigee

To obtain the normal print at the crossing of the ascending node, insert a fixed point number in:

30342 CCNT DEC .... Ascending node print\*

44214 CANT DEC . . . . . Apogee print\*

44215 CPNT DEC . . . . . Perigee print\*

The number determines how often the print operates, i.e., 2 means print every second crossing of the ascending node. The print occurs at end of the first integration interval following the crossing.

O. Ordinarily the Printout Includes the Following:

Time, position with respect to reference body.

Velocity with respect to reference body.

More printout may be called for by putting a number (fixed point) into

1 in KMVEC+2 adds vehicle position with respect to earth

1 in KMVEC+3 adds vehicle position with respect to moon

1 in KMVEC+4 adds moon's position with respect to earth

1 in KMVEC+5 adds vehicle position with respect to sun

1 in KMVEC+6 adds vehicle position with respect to Venus

<sup>1</sup> in KMVEC+7 adds vehicle position with respect to Mars

<sup>1</sup> in KMVEC+8 adds vehicle position with respect to Jupiter

## Fixed point numbers

- 1 in KMVEC+9 adds perturbation displacement
- 1 in KMVEC+10 adds perturbation velocity
- 1 in KMVEC+11 adds perturbation acceleration

In addition, printout will always contain

14517 LLDEC+2	RA and declination
14715 EAZCAL-5	Coordinates of subsatellite point
14760 1CONV+5	Moon longitude and latitude in moon reference
14765 PREXT	Osculating elements

These prints may be eliminated with the following cards:

14517 LLDEC+2	OCT 002000014521	TRA *+2
14715 EAZCAL-5	OCT 002000014717	TRA *+2
14760 1CONV+5	OCT 002000014762	TRA *+2

To restore:

14517 LLDEC+2	OCT 007400421321	TSX LLACP, 4
14715 EAZCAL-5	OCT 007400421300	TSX PPRINT, 4
14760 1CONV+5	OCT 007400421342	TSX LLMNP, 4

P. Osculating Elements Print is normally given at each print time.

To revoke:

14765 PREXT	OCT 002000014770	TRA MOOSCO
-------------	------------------	------------

To restore:

14765 PREXT	OCT 007400425163	TSX ELCO, 4
-------------	------------------	-------------

Q. The Normal Output Refers the Osculating Elements to the Equatorial Plane with the x-axis along the mean equinox of date.

The modification described below will trigger, in addition, the printing of the inclination, ascending node and argument of perigee with respect

to the moon's orbital plane and the x-axis along the moon-earth vector, either at the running time  $t$  (see Figure 3) or at an arbitrary fixed time  $t_1$ . In addition, this modification will print the instantaneous osculating elements of the moon's orbit with respect to the equatorial plane.

26015 ELPT OCT 002000034153 TRA INSMOO

To use orbital plane at time  $t_1$ :

26015 ELPT OCT 002000034127 TRA TOOSC+1  
34126 TOOSC DEC .... time (1)\*\*

(1) If no number is loaded into TOOSC, the orbital plane at the initial time ( $t = 0$ ) is used.

To revoke:

26015 ELPT OCT 076100000000 NOP

---

\*\* Floating point numbers

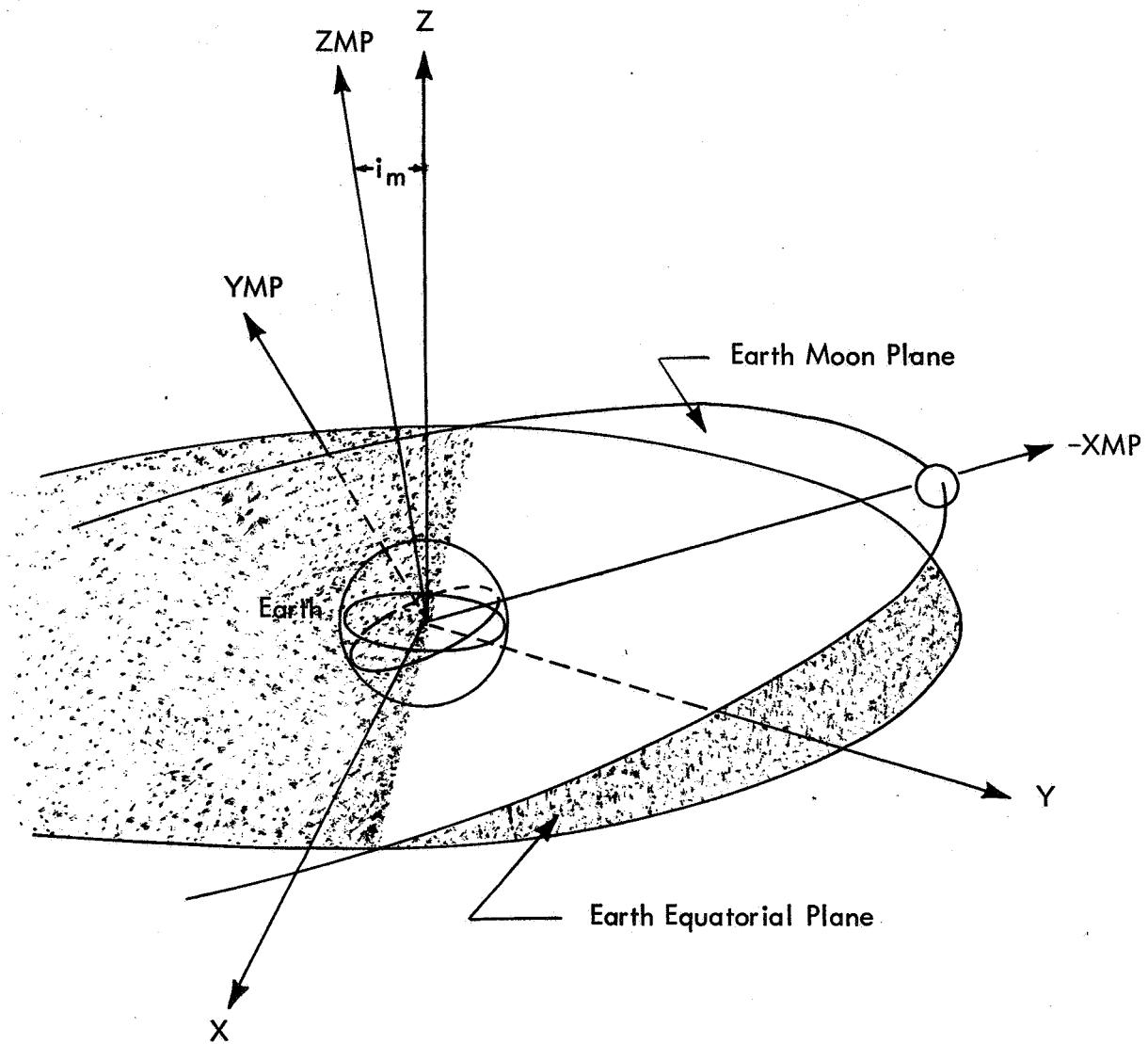


Figure 3. Instantaneous Lunar Plane Coordinates

#### R. Moon Fixed and Rotating Coordinate System

Vehicle position is provided in two coordinate systems, one rotating and one space fixed, based on the earth-moon plane. For both systems, the origin is at the center of the earth with the positive x-axis directed toward the center of the moon. The x y plane is the plane of the moon's orbit about the earth, and the z axis is in a northerly direction so as to form a right-hand system. For the rotating system (XROT, YROT, ZROT) the x-axis moves with

the moon; for the space fixed system (SINJ, YINJ, ZINJ) the x-axis is directed toward the position of the moon's center at injection.

To use, insert the following modification card:

25626 DONRCO OCT 076100000000 NOP

To revoke:

25626 DONRCO OCT 002000025707 TRA FOSCL

S. To Print the Coordinates of the Earth Sub-Satellite point and right ascension and declination in other than earth or moon reference, the following card must be added:

Modification card:

14476 NESS OCT 076100000000 NOP

To revoke:

14476 NESS OCT 010000014723 TZE SUBSEX

T. Floating Point Spill Print

A routine is included which will make the proper correction for floating point spills. Normally this routine will print as a warning

OVERFLOW IN LOC .....

To eliminate print, use the following card:

25006 FPSP OCT 002000025010 TRA \*+2

To restore:

25006 FPSP OCT 007400403747 TSX E\$WOT, 4

Overflow Diagnostic

This optional debugging feature prints at each overflow the contents of the index registers, AC, MQ, the address part of the three instructions preceding the overflow and the numbers in the listed locations.

To activate, use the following card:

25010 OFLDG	OCT 076100000000	NOP
-------------	------------------	-----

To revoke:

25010 OFLDG	OCT 002000025052	TRA LLXT
-------------	------------------	----------

**U. Error Return**

If an error is detected the program has three options:

1. ERPRT The octal location of the error is written on line and off line. The normal end dump is written on A3 and the program goes to the next case.
2. ERPRTS The program behaves as above except that it requests a memory dump and halts.
3. ERPRTC The program behaves as in ERPRT except that it will continue the current case through three of these errors before going on to next case.

**V. The Flight Path Angle and Azimuth referred to both the geodetic and geocentric horizontal planes in earth reference are printed normally.**

To eliminate:

14722 EAZCAL	OCT 076100000000	NOP
--------------	------------------	-----

To revoke:

14722 EAZCAL	OCT 007400436120	TSX AZZCAL, 4
--------------	------------------	---------------

**In Moon Reference**

To eliminate:

14762 MAZCAL	OCT 076100000000	NOP
--------------	------------------	-----

To revoke:

14762 MAZCAL	OCT 007400436120	TSX AZZCAL, 4
--------------	------------------	---------------

W. Additional Stopping Conditions were introduced for moon satellites when the radius from the moon is too large, or too small. These distances may be changed by the following cards:

Modification card:

30226 MRVEC	DEC .....	Minimum distance** (.27 ER currently)
30227 MRVEC+1	DEC .....	Maximum distance** (10. ER currently)

X. Shadow Calculation

The time boundaries defining the umbra and penumbra are determined (including refraction) by a linear interpolation across the two nearest integration steps. The radiation pressure switch is controlled only to the first integration step after a change occurs. Subsequent updates will include an optional refinement to integrate exactly to the boundary.

The entry, exit, time in and accumulated time for sunlight, umbra and penumbra are printed at each change. The following card activates shadow testing and printing:

15334 SHAENT	OCT 007400426171	TSX WSHADE, 4
--------------	------------------	---------------

To revoke:

15334 SHAENT	OCT 076100000000	NOP
--------------	------------------	-----

Y. Polar Scale Factor is used to change the units of input quantities (for polar input only).

The modification cards are:

30145 ICONT	DEC .....	(1)**
30146 POLSC	DEC .....	(2)**

- (1) Number of time units in one hour.
- (2) Number of distance units in one earth radius.

\*\* Floating point numbers

To revoke:

30145 ICNT DEC 3600., 6378.165 \*\*

Z. Ecliptic Coordinate Output

This gives additional output (Section X-B-11) referenced to the ecliptic plane.

To initiate, include:

26016 ECELPT OCT 002000034740 TRA ECLCOO

To revoke:

26016 ECELPT OCT 076100000000 NOP

A-1. Impact Parameter Calculation (Appendix U)

This calculates miss distances on the impact plane.

To use, include:

12021 TIMPPS OCT 076100000000 NOP

To revoke:

12021 TIMPPS OCT 002000012074 TRA ETIMPP

If several cases are stacked, the same impact plane is used for all of them. If a new impact plane is desired, include the card:

35660 NOMTR DEC 0

B-1. Geocentric Velocity and Geodetic Position Input

To use, include:

12310 LBASC-4 OCT 007400412317 TSX BASCOR, 4

To revoke:

12310 LBASC-4 OCT 007400412320 TSX POLCAR, 4

\*\* Floating Point Numbers

### C-1. Trajectory Search

The search program is used in connection with the polar load and is activated in two ways:

1. If the search consists of circumlunar trajectories with return to earth

15204 ITENT	OCT 002000041307	TRA STIT
44001 IELCO	DEC x, x, x, x	x=1 or 0*
DEC 1, 0, 0, 0	Outgoing miss parameter only	
DEC 0, 1, 0, 0	Return miss parameter only	
DEC 1, 1, 0, 0	Both miss parameters	
DEC 0, 0, 1, 0	Outgoing osculating elements near perigee	
DEC 0, 0, 0, 1	Return osculating elements near perigee	
DEC 0, 0, 1, 1	Both osculating elements	
etc.		

2. If the search is planetary

15205 VITENT	OCT 002000042334	TRA VIT
--------------	------------------	---------

The search input is also loaded through modification cards, a sample of which follows:

43224 MAXIT	DEC 5	(maximum number of iterations desired)*
42603 IVAR	DEC 0, 0, 10., .0001, 0, 0, 0**	

Input quantities to be varied in the units and sequence of the polar load, i.e., latitude, longitude, altitude, velocity, azimuth, flight path angle and time, and the amounts by which to vary the inputs to get the partial derivatives are loaded. In the sample given, vary height 10 km, speed .0001 km/sec, and keep all others constant. Next

42614 LQUANT	DEC 5, 10*	
--------------	------------	--

the code of the quantities to be achieved. In the sample, pericynthion and perigee. The quantity code is as follows:

---

\* Fixed point numbers

\*\* Floating point numbers

1. Lunar inclination	(degrees)
2. Lunar ascending node	(degrees)
3. Lunar argument of pericynthion	(degrees)
4. Lunar time of pericynthion	(hours)
5. Lunar pericynthion radius	(km)
6. Earth inclination	(or planet)
7. Earth ascending node	(or planet)
8. Earth argument of perigee	(or planet)
9. Earth time of perigee	(or planet)
10. Earth perigee radius	(or planet)
11. B·T - Miss parameter	(planet or moon)
12. B·R - Miss parameter	(planet or moon)

Next

42626 QUANT      DEC 6000., 5000.\*\*

are the desired values of the dependent variables. These must be listed in the same order as LQUANT. Therefore, these are pericynthion and perigee radius (in km), respectively.

Finally, the tolerances on the above quantities should be given and again in the same order as the preceding two, i.e., LQUANT and QUANT

42640 ICONV      DEC 10., 100.\*\*

also in km. Thus the tolerance on pericynthion radius is 10 km and for perigee 100 km. If the solution converges to within the specified tolerance, the iterations will stop.

A maximum of any seven dependent variables can be selected. All of the search input modifications must be repeated for each repeat or stacked case with appropriate changes as desired.

To repeat - the sample input for search is as follows:

15204	OCT 002000041307	ACTIVATE SEARCH
44001	DEC 1, 1, 1, 1	HELCO*
43224	DEC 5	MAXIT*

---

\* Fixed point numbers

\*\* Floating point numbers

42603	DEC 0, 0, 10., .0001, 0, 0, 0	IVAR**
42614	DEC 5, 10	LQUANT*
42626	DEC 6000., 5000.	QUANT**
42640	DEC 10., 100.	ICONV**

The normal input for the trajectory search is by modification cards. The lunar search option normally iterates on osculating elements referred to the equatorial system. To iterate on elements referred to the earth-moon plane, use the following card:

41360 EMP6	OCT 076100000000	NOP
To revoke:		
41360 EMP6	OCT 002000041404	TRA EQCOO

D-1. Ephemeris Time

The planetary coordinates are interpolated using ephemeris time

$$ET = UT + \Delta T$$

An approximate value of  $\Delta T$  (35 sec) is used. To change, use the following card, giving  $\Delta T$  in hours.

30164 ETMUT	DEC .....	$(\Delta T$ in hrs)**
-------------	-----------	-----------------------

To revoke:

30164 ETMUT	DEC .009888888889	$(\Delta T$ is 35 sec)**
-------------	-------------------	--------------------------

E-1. If it is desirable to include additional descriptive material in the title printout, the following locations may be used:

21070 TTPRN	BCD	Descriptive information
21102	BCD	Descriptive information
20 symbols (2 cards) maximum.		

The first letter of the first card should be in column 14. The first letter of the second card may be in column 13.

---

\* Fixed point numbers

\*\* Floating point numbers

F-1. Beta Integration

The program has an option for integrating Beta instead of time. Computation is much faster in this mode but printout doesn't occur at exact times. Integration intervals have to be chosen with care.

Modification card: For Beta Mode

40537 BETRIG OCT 076100000000 NOP

To revoke:

40537 BETRIG OCT 002000000000 TRA

G-1. Apogee, Perigee and Nodal Crossing Prints are activated by the following modification cards:

30342 CCNT DEC n \*

44214 CANT DEC n \*

44215 CPNT DEC n \*

CCNT prints every  $n^{\text{th}}$  nodal crossing

CANT controls apogee prints

CPNT controls perigee prints

## IX. INPUT

This section contains the following information:

- A. General inputs necessary for running a case,
- B. Stacking cases.
- C. Sample input for polar coordinates, no modifications.
- D. Sample input for Cartesian coordinates, no modifications.
- E. Sample input with modifications.
- F. Radar input information.

### A. General Inputs

<u>Title and Units</u>	<u>See Note</u>
Modification cards, if any	(1)
TRA 2, 4	(2)
DEC 0, 1, 2, 3 or 4	(3)*
0 - Cartesian coordinate input	
1 - Equatorial polar coordinate input	
2 - Translunar plane input	
3 - Ecliptic polar coordinate input	
4 - Osculating element input	
DEC 4, 5 or 6	(4)*
4 - Venus	
5 - Mars	
6 - Jupiter	

are tested for possible reference planets during the flight and one must be designated. Moon, Sun or Earth are tested automatically and need not be designated.

\*Fixed point numbers,

<u>Load Number</u>	<u>Cartesian Coordinates</u>	<u>Polar Coordinates</u>	<u>Translunar Plane</u>	<u>Element Load</u>	<u>See Note</u>
- 1	Omit	Latitude - Degrees and decimals	Lead angle $\Psi$ - Degrees and decimals	1, 2, 3 or 4*	(3)**
0		Case number			(5)*
1	x - km	Longitude - Degrees and decimals	Argument of radius - Degrees and decimals	Argument of perigee	(3)**
2	y - km	Altitude - km	Altitude - km	RA of ascending node	(3)**
3	z - km	Speed - km/sec	Speed - km/sec	Inclination	(3)**
4	$\dot{x}$ - km/sec	Flight path azi- muth - Degrees and decimals	Inclination of translunar plane - Degrees and decimals	Semi-major axis	(3)**
5	$\dot{y}$ - km/sec	Flight path angle- Degrees and decimals	Flight path angle - Degrees and decimals	Eccentricity	(3)**
6	$\dot{z}$ - km/sec	Longitude of vernal equinox- Degrees and decimals or indicator	Immaterial	Uses number from (-1)  1 - Time of perigee 2 - Mean Anomaly 3 - Eccentric Anomaly 4 - True Anomaly	(3)(15)**

\* Fixed point numbers

\*\* Floating point numbers

<u>Load Number</u>	<u>Title and Units</u>	<u>See Note</u>
7	Launch time - days	(8)(6)**
8	Launch time - hours	(8)**
9	Launch time - minutes	(8)**
10	Launch time - seconds	(8)**
11	Initial print suppress - hours	(7)**
12	Year of launch	(17)*
13	Initial reference - origin indicator 1 - Earth                           4 - Venus 2 - Moon                           5 - Mars 3 - Sun                           6 - Jupiter	(3)(9)*
14	Thrust indicator 0 - No thrust 1 - Thrust	(10)*
15	Maximum time of flight - hours	(11)**

If thrust is used, the number 1 has to be loaded into "Load+14" and additional input data have to be supplied. These cards are inserted following the regular input, but preceding the radar input, if any. The thrust load has the following format:

T0	Integration interval to be used during thrust period	(16)**
T1	Number of thrust periods, 24 maximum	(18)*
T2	Initial mass of vehicle - pounds	(19)**
T3	Mass flow-pound mass/pound thrust/hour	**
T4	Initial time of first thrust period	**

\*Fixed point numbers.

\*\*Floating point numbers.

For each thrust period:

1T1	$T_x$	x-component of thrust - pounds	**
1T2	$T_y$	y-component of thrust - pounds	**
1T3	$T_z$	z-component of thrust - pounds	**
1T4	0, 0	Space reserve	
1T5	$t_f$	Termination time of this thrust period	**

If simulated radar information is desired, the following input must be supplied: (See VIII-C)

(12)

R0	Number of stations	(13)*
R1	BCD 4 - Station name	(5)(14)
R2	Longitude - degrees, minutes, seconds	(12)(14)**
R3	Latitude - degrees, minutes, seconds	(12)(14)**
R4	Altitude - feet	(12)(14)**

NOTES:

1. See Section VIII for modifications available.
2. Terminates reading of modification cards. This card must always be included.
3. See Sections IV and VIII and translunar plane section.
4. The choice of planet is significant only if trajectories are to be computed which will come close to either Mars or Venus or Jupiter. In this case, the planet designated may serve as a reference body if its sphere of influence is entered. For all other trajectories, the designation of a planet only affects the output and is usually unimportant. A choice, however, must be made.

\* Fixed point number

\*\* Floating point numbers

NOTES: (cont.)

5. These inputs are used only for identification and not for computing purposes.
6. Number of days since December 31, 0.0 hrs UT. (See footnote 1 of Section IV.)
7. This suppresses printing until after specified time has been exceeded. The input time is arbitrary and may be chosen as desired.
8. The number of days must be a floating point integer, the minutes and seconds may be included as a decimal fraction of hours. In this case, 0 must be entered on cards 9 and 10.
9. Indicates the body whose center is the origin of the initial reference system.
10. For a more detailed description of the use of the thrust programs, see Section IX-A, B (Input).
11. See Section V.
12. See Section VII.
13. Enter number of tracking stations.
14. Enter the following information for each station: (See Section VII)
  - BCD 4 Station name, 24 symbols maximum
  - DEC Longitude
  - DEC Latitude
  - DEC Altitude

Longitude and latitude may be given in degrees and decimals if more convenient. 0 then has to be used in place of minutes and seconds. The resulting printout of the polar station coordinates will then show the integral number of degrees only. The station position in core, however, will reflect the fractional part.

15. For polar coordinate input only. If zero is loaded in this position, the load in position 1 "longitude" is interpreted as right ascension. If a fixed point 1 is loaded, the program computes the longitude of the vernal equinox at launch time from the data supplied on cards 7, 8, 9 and 10. If for some reason this computation is not desired, a floating point number is loaded on card 6 and then this number will be used as the longitude of the vernal equinox.

NOTES: (cont.)

16. If fractional parts of hours are used for integration intervals, it is recommended that multiples of negative powers of 2 are used to eliminate round-off error in the time.
17. This quantity ensures that the proper file on the planetary tape is used. It will stop the program if the wrong tape is mounted or the proper file cannot be found.
18. Periods of no thrust intervening between thrust periods must be included in this count.
19. No staging is considered, but may be included if required.
20. For a period of no thrust intervening between two thrust periods, cards 1T1 through 1T4 have to be included with 0 in place of  $T_x$ ,  $T_y$  and  $T_z$ . In each case the final time  $t_f$  is used as initial time for the next thrust period, until the indicated number of thrust periods has been reached.

B. Stacking of Cases

If many cases are to be run differing only by a few parameters, it is not necessary to repeat that part of the input which remains unchanged. The changes only are to be loaded but an octal location must be punched in columns 2 through 6. For instance, if only the latitude is to be changed, the following card is loaded:

31263                    DEC .....                    latitude

The table on the following page gives a complete list of the load and their octal locations.

If the launch time is changed in a polar load stack, the computation of the longitude of the vernal equinox must be re-initiated by loading 1 into LOAD+6.

These cards are followed by three TRA 2, 4 to terminate loading. If radar is used and the station coordinates are unchanged, an "Avoid Radar Load" card (described in Section VIII-C) has to be included. If thrust is used, the thrust load has to be repeated or the following modification card has to be included to cancel the thrust loading:

TABLE 1

LOAD LOCATIONS

<u>Load Number</u>	<u>Octal Location</u>	<u>Title</u>
- 1	31263	latitude
0	4	case number
1	5	x or longitude
2	6	y or height
3	7	z or speed
4	31270	x or azimuth
5	1	y or flight path angle
6	2	z or longitude of vernal equinox
7	3	days
8	4	hours
9	5	minutes
10	6	seconds
11	7	print suppress
12	31300	year
13	1	origin indicator
14	2	thrust indicator
15	3	maximum time

12630 THRST+2

OCT 076100000000

NOP

To revoke:

12630 THRST+2

OCT 007400420407

TSX THLD, 4

C. Sample Input for Polar Coordinates - with repeat case

30153 DEC 1	TESSTR - modification to include tesseral harmonics	
21070 BCD	FRED SHAFFER	
TRA 2, 4	Go to regular load - end of modification cards	
DEC 1	Polar coordinates	
DEC 5	Mars is planet choice	
DEC 30.60860	Latitude in degrees and decimals	- 1
DEC 1	Case number	0
DEC - 71.78139	Longitude in degrees and decimals	1
DEC 160.72594	Altitude in kilometers	2
DEC 7.8439280	Velocity in km/sec	3
DEC 77.89589	Flight path azimuth in degrees and decimals	4
DEC -.0317	Flight path angle in degrees and decimals	5
DEC 1	Program computes longitude of vernal equinox at launch time	6
DEC 268.	Launch time in days from Dec. 31, 0.0h UT	7

DEC 14.	Launch time in hours from midnight UT	8
DEC 0.	Launch time in minutes	9
DEC 0.	Launch time in seconds	10
DEC 0.	Print suppress in hours	11
DEC 1962	Year of launch	12
DEC 1	Earth is initial reference origin	13
DEC 0	No thrust	14
DEC 24.	Maximum time of flight in hours	15
30153 DEC 0	No tesseral harmonics in repeat case	
21102 BCD	NO TESSERAL HARMONICS	
TRA 2, 4		
TRA 2, 4		
TRA 2, 4		
TRA 37777	End of run, non-monitor	

D. Sample Input for Cartesian Coordinates - no modifications

TRA 2, 4		
DEC 0	Cartesian coordinates	
DEC 4	Venus is planet choice	
DEC 1	Case number	0
DEC 4953.3445	x-coordinate in km	1
DEC 3716.7623	y-coordinate in km	2
DEC 2224.0628	z-coordinate in km	3

DEC 7.1423107	x-component in km/sec	4
DEC 7.6362243	y-component in km/sec	5
DEC 3.1457277	z-component in km/sec	6
DEC 33.	Launch time in days	7
DEC 2.	Launch time in hours	8
DEC 59.	Launch time in minutes	9
DEC 42.72	Launch time in seconds	10
DEC 0.	Print suppress in hours	11
DEC 1966	Year of launch	12
DEC 1	Earth is initial reference origin	13
DEC 0	No thrust	14
DEC 168.	Maximum time of flight in hours	15
TRA 37777	End of data deck, non-monitor	

E. Sample Input for Cartesian Coordinates - with modifications

12654 OCT 007400423630 Load radar data  
 14771 OCT 007400424002 Compute radar output  
 25006 OCT 002000025010 Eliminate overflow print  
 21070 BCD IMP ORBIT WITH RADAR  
 30177 DEC 4.5 Change near, far earth boundary  
 TRA 2, 4  
 DEC 0 Cartesian coordinates  
 DEC 5 Mars is planet choice

DEC 102	Case number	0
DEC 6336.4095	x-coordinate in km	1
DEC 812.60758	y-coordinate in km	2
DEC 1600.4426	z-coordinate in km	3
DEC .12118217	$\dot{x}$ -component in km/sec	4
DEC 9.4967498	$\dot{y}$ -component in km/sec	5
DEC -5.3016564	$\dot{z}$ -component in km/sec	6
DEC 152.	Launch time in days	7
DEC 11.	Launch time in hours	8
DEC 3.	Launch time in minutes	9
DEC 58.	Launch time in seconds	10
DEC 0	Print suppress in hours	11
DEC 1963	Year of launch	12
DEC 1	Earth is initial reference origin	13
DEC 0	No thrust	14
DEC 160.	Maximum time of flight in hours	15

F. Radar Input Information

DEC 1	Number of stations	R0
BCD	4 MID ATLANTIC (station name)	R1
DEC -30., 0., 0.	Longitude of station - degrees, minutes, seconds	R2
DEC 5., 0., 0.	Latitude of station - degrees, minutes, seconds	R3
DEC 0.0	Altitude of station - feet	R4
TRA 37777	End of data deck, non-monitor	

## X. OUTPUT

### A. Program Outputs

The following information is printed as the output of the program.

1. Modification cards
2. Title
3. Case number and any identifying titles
4. Launch time - days, hours, minutes, seconds
5. Input data - printed out exactly as they entered the program
6. Octal locations of LOAD-1 through LOAD+7
7. List of parameters used in run
8. At each rectification the following data are printed:

RECTIFICATION PRINT       (a)       REFERENCE

      (b)       PERT OVER UNPERT =       (c)      

AT TIME =       (d)       DELTA T =       (e)      

(a) Reference body

(b) and (c) indicate the reason for rectification:

If (c) = 0 then reference body has been changed as indicated by (b) and the following nomenclature is used to indicate the new reference body:

MR - Moon reference

VR - Venus reference

SR - Sun reference

MA - Mars reference

ER - Earth reference

JR - Jupiter reference

If (c) ≠ 0 then the position, velocity, acceleration perturbations or the incremental eccentric anomaly have exceeded the permissible limits and (b) indicates which has been exceeded (see Section VI). These indications are given as:

PO - Position

VL - Velocity

AC - Acceleration

TH - Incremental eccentric anomaly

(d) Rectification time in hours from time of launch

(e) Integration interval to be used



OSCULATING ELEMENTS AT TIME T = \_\_\_\_\_

TRUE ANOMALY = \_\_\_\_\_ (n)

SEM MAJ AXIS = \_\_\_\_\_ (o)

ECCENT = \_\_\_\_\_ (p)

PERICENT = \_\_\_\_\_ (q)

APOCENT = \_\_\_\_\_ (r)

INCLINATION = \_\_\_\_\_ (s)

(j) Moon longitude - Angle between the projection of the vector from the moon to the vehicle onto the moon's orbital plane and the moon-earth vector (moon reference only) - deg

(k) Moon latitude - Angle between the radius vector connecting the moon and the vehicle and its projection onto the orbital plane of the moon about the earth (moon reference only) - deg

(l) Selenocentric flight path azimuth - deg

(m) Selenocentric flight path angle - deg

(n) True anomaly - deg

(o) Semi-major axis of trajectory - ER

positive = ellipse

negative = hyperbola

(p) Eccentricity of trajectory\*\*

(q) Closest distance to the reference body (not necessarily the earth) - km\*\*

(r) Farthest distance from the reference body (not necessarily the earth) - km\*\* (Meaningful only for elliptic orbits.)

(s) Inclination of the orbital plane defined as the angle between the positive polar axis and the angular momentum vector - deg\*\*

\*\* These are the osculating values and hence only constitute an estimate of the quantities described.

ARG PERIC	= _____	(t)
PERIOD	= _____	(u)
MEAN MOT	= _____	(v)
R A ASC NODE	= _____	(w)
M ANOMALY	= _____	(x)
E ANOMALY	= _____	(y)
T PERIC	= _____	(z)
UNIT PERICENTER POSITION VECTOR	= _____	(aa)
UNIT ANGULAR MOMENTUM VECTOR	= _____	(bb)

- (t) Argument of pericenter - Angle measured from the ascending node to the pericenter vector - deg\*\*  
Set to zero for circular orbits and poorly determined for near-circular orbits
- (u) Period - hrs\*\*
- (v) Mean motion - rad/hr\*\*
- (w) Right ascension of the ascending node measured from the vernal equinox eastward along the equator - deg\*\*
- (x) Mean anomaly - rad\*\*
- (y) Eccentric anomaly - rad\*\*
- (z) Time of nearest pericenter - hrs\*\*
- (aa) Components of the unit vector directed from reference toward pericenter\*\*
- (bb) Components of the unit angular momentum vector

\*\* Osculating values

B. Optional Outputs

1.  $XVE = \underline{\hspace{2cm}}$   $YVE = \underline{\hspace{2cm}}$   $ZVE = \underline{\hspace{2cm}}$   $RVE = \underline{\hspace{2cm}}$  (a)  
 $XVM = \underline{\hspace{2cm}}$   $YVM = \underline{\hspace{2cm}}$   $ZVM = \underline{\hspace{2cm}}$   $RVM = \underline{\hspace{2cm}}$  (b)  
 $XME = \underline{\hspace{2cm}}$   $YME = \underline{\hspace{2cm}}$   $ZME = \underline{\hspace{2cm}}$   $RME = \underline{\hspace{2cm}}$  (c)  
 $XVS = \underline{\hspace{2cm}}$   $YVS = \underline{\hspace{2cm}}$   $ZVS = \underline{\hspace{2cm}}$   $RVS = \underline{\hspace{2cm}}$  (d)  
 $XVVN = \underline{\hspace{2cm}}$   $YVVN = \underline{\hspace{2cm}}$   $ZVVN = \underline{\hspace{2cm}}$   $RVVN = \underline{\hspace{2cm}}$  (e)  
 $XVMR = \underline{\hspace{2cm}}$   $YVMR = \underline{\hspace{2cm}}$   $ZVMR = \underline{\hspace{2cm}}$   $RVMR = \underline{\hspace{2cm}}$  (f)  
 $XVJP = \underline{\hspace{2cm}}$   $YVJP = \underline{\hspace{2cm}}$   $ZVJP = \underline{\hspace{2cm}}$   $RVJP = \underline{\hspace{2cm}}$  (g)  
 $XI = \underline{\hspace{2cm}}$   $ETA = \underline{\hspace{2cm}}$   $ZETA = \underline{\hspace{2cm}}$   $PERT = \underline{\hspace{2cm}}$  (h)  
 $XIDT = \underline{\hspace{2cm}}$   $ETADT = \underline{\hspace{2cm}}$   $ZETADT = \underline{\hspace{2cm}}$   $VPERT = \underline{\hspace{2cm}}$  (i)  
 $D2XI = \underline{\hspace{2cm}}$   $D2ETA = \underline{\hspace{2cm}}$   $D2ZETA = \underline{\hspace{2cm}}$   $APERT = \underline{\hspace{2cm}}$  (j)

The above optional output appears between XRDT and right ascension in the standard output. For instructions on how to obtain, see Section VIII-O.

- (a) Coordinates of vehicle with respect to the earth - km
- (b) Coordinates of vehicle with respect to the moon - km
- (c) Coordinates of moon with respect to the earth - km
- (d) Coordinates of vehicle with respect to the sun - km
- (e) Coordinates of vehicle with respect to Venus - km
- (f) Coordinates of vehicle with respect to Mars - km
- (g) Coordinates of vehicle with respect to Jupiter - km
- (h) Perturbation vector and magnitude of the perturbations with respect to the reference body - km
- (i) Perturbation velocity vector and magnitudes - km/sec
- (j) Perturbation acceleration vector and magnitudes - km/sec<sup>2</sup>

## 2. Moon Rotating and Fixed Coordinate Systems

XINJ = \_\_\_\_\_ YINJ = \_\_\_\_\_ ZINJ = \_\_\_\_\_ RINJ = \_\_\_\_\_ (a)

XROT = \_\_\_\_\_ YROT = \_\_\_\_\_ ZROT = \_\_\_\_\_ RROT = \_\_\_\_\_ (b)

XEM = \_\_\_\_\_ YEM = \_\_\_\_\_ ZEM = \_\_\_\_\_ REM = \_\_\_\_\_ (c)

- (a) Coordinates of the vehicle with respect to a fixed coordinate system defined at injection (see Appendix Q) - km
- (b) Coordinates of the vehicle with respect to a rotating coordinate system (see Appendix Q) - km
- (c) Coordinates of the earth with respect to the moon - km

## 3. Moon Osculating Elements

REFERENCE PLANE IS MOON ORBITAL PLANE AT (a)

TIME = \_\_\_\_\_

INCLINATION = \_\_\_\_\_ (b)

ASC NODE = \_\_\_\_\_ (c)

ARG PERICENT = \_\_\_\_\_ (d)

CHNG ASC NODE = \_\_\_\_\_ (e)

INCL MOON PLANE = \_\_\_\_\_ (f)

RA ASC NODE OF MOON = \_\_\_\_\_ (g)

ARG OF LAT OF MOON = \_\_\_\_\_ (h)

- (a) Time in hours from launch. The reference plane is the osculating orbital plane of the moon. The x-axis is along the moon-earth vector. The reference system is variable or fixed, depending on the option selected (see Section VIII-Q).
- (b) Inclination of vehicle plane with respect to the orbital plane of the moon - deg

- (c) Ascending node - Angle between the nodal line and x-axis - deg
- (d) Argument of pericenter - deg
- (e) Change in ascending node from time zero - deg
- (f) Inclination of the moon orbital plane with respect to the earth's equatorial plane - deg
- (g) Right ascension of the ascending node of the moon - deg
- (h) Argument of latitude of the moon - angle between the moon's position vector and the ascending node of the moon - deg

4. Angle from Ascending Node to Vehicle

ANGLE FROM ASCEND NODE TO SAT = \_\_\_\_\_ (a)

- (a) Angle from the ascending node to the satellite - deg

5. Error Print

ERROR IN LOC \_\_\_\_\_ (a)

- (a) Octal location subsequent to where an error occurs.

6. Floating Point Spill Print

OVERFLOW IN LOC \_\_\_\_\_ (a)

- (a) Octal location of the instruction which caused a floating point spill to occur.

7. End Print

Four blocks of storage that are useful in checking and debugging.

8. Shadow Print

PASSAGE FROM  $\left\{ \begin{array}{l} \text{SHADOW} \\ \text{PENUMBRA} \\ \text{SUN} \\ \text{PENUMBRA} \end{array} \right\}$  TO  $\left\{ \begin{array}{l} \text{PENUMBRA} \\ \text{SHADOW} \\ \text{PENUMBRA} \\ \text{SUN} \end{array} \right\}$

AT (a) TIME IN  $\left\{ \begin{array}{l} \text{SHADOW} \\ \text{PENUMBRA} \\ \text{SUN} \\ \text{PENUMBRA} \end{array} \right\}$  (b) ACCUMULATED TIME (c), (d)

- (a) Time at which vehicle crosses denoted shadow boundary - hrs
- (b) Total time the vehicle spends in denoted shadow region during current traverse - hrs
- (c) Total accumulated time spent in denoted shadow region since launch - hrs
- (d) 0 indicates earth's shadow  
1 indicates moon's shadow

9. Radar Output

STATION	(a)
AZIMUTH	(b)
ELEVATION	(b)
TOPOC. RA	(c)
TOPOC. DECL.	(c)
SLT RNG	(d)
RANGE	(e)

- (a) Station name (identification) for each station
- (b) Azimuth and elevation with respect to each station - deg
- (c) Topocentric right ascension and declination with respect to each station - deg
- (d) Slant range to each station - km
- (e) Rate of change of slant range for each station - km/sec

If the elevation is negative (vehicle is below horizon), this print is suppressed for the station of reference.

10. Flight Path Azimuth and Angle Output

GEOCENTRIC AZIMUTH = \_\_\_\_\_ (a)  
ELEVATION = \_\_\_\_\_ (b)  
GEOD AZIM = \_\_\_\_\_ (c)  
ELEV = \_\_\_\_\_ (d)

- (a) Geocentric flight path azimuth - deg
- (b) Geocentric flight path angle - deg
- (c) Geodetic flight path azimuth - deg
- (d) Geodetic flight path angle - deg

11. Ecliptic Coordinate Output

EXES = \_\_\_\_ EYES = \_\_\_\_ EZES = \_\_\_\_ ERES = (a) ELONG = (b)  
EXTS = \_\_\_\_ EYTS = \_\_\_\_ EZTS = \_\_\_\_ ERTS = (c) TLONG = (d)  
EXVS = \_\_\_\_ EYVS = \_\_\_\_ EZVS = \_\_\_\_ ERVS = (e) VLONG = (f)  
XECL = \_\_\_\_ YECL = \_\_\_\_ ZECL = \_\_\_\_ RECL = (g)  
DXECL = \_\_\_\_ DYECI = \_\_\_\_ DZECL = \_\_\_\_ DRECL = (h)

- (a) Ecliptic coordinates of earth with respect to the sun - km
- (b) Ecliptic longitude of earth - deg
- (c) Ecliptic coordinates of planet with respect to the sun - km
- (d) Ecliptic longitude of planet - deg
- (e) Ecliptic coordinates of vehicle with respect to the sun - km
- (f) Ecliptic longitude of vehicle - deg
- (g) Ecliptic coordinates of vehicle with respect to reference body - km
- (h) Ecliptic velocity of vehicle with respect to reference body - km/sec

ANGLE BETWEEN RVE-RVS = (i) (j)

ELEMENTS REFERRED TO ECLIPTIC PLANE AT TIME T = \_\_\_\_\_

ECC = \_\_\_\_\_ (k)

INCL	=	_____	(l)
ASCN	=	_____	(m)
ARPG	=	_____	(n)
LONG RREF	=	_____	(o)
LAT RREF	=	_____	(p)
AZIMUTH	=	_____	(q)
ELEV	=	_____	(r)

- (i) Angle between vehicle-earth and vehicle-sun vectors - deg
- (j) Angle between vehicle-planet and vehicle-sun vectors - deg
- (k) Eccentricity of trajectory
- (l) Inclination of vehicle plane with respect to the ecliptic plane - deg
- (m) Ascending node - angle between nodal line and x-axis - deg
- (n) Argument of perigee - deg
- (o) Ecliptic longitude of vehicle with respect to the reference body - deg
- (p) Ecliptic latitude of vehicle with respect to the reference body - deg
- (q) Flight path azimuth with respect to the reference body on celestial sphere - deg
- (r) Flight path angle with respect to the reference body on celestial sphere - deg

## 12. Reentry Output

REENTRY PRINT	TIME	INERTIAL SPEED (km/sec)
---------------	------	-------------------------

Right ascension, declination, earth subsatellite points, and flight path azimuth and angle as given above.

## 13. Trajectory Search Output

The output consists of the normal ITEM output for a nominal trajectory and the same trajectory output for each variation requested. The output format used only for the trajectory search follows:

VARIATION IN INITIAL CONDITIONS	<u>(a)</u>	<u>(b)</u>	<u>(c)</u>	<u>(d)</u>	<u>(e)</u>	<u>(f)</u>	<u>(g)</u>
QUANTITY CODE	<u>(h)</u>						
DESIRED VALUES OF ABOVE QUANTITIES	<u>(i)</u>						
REQUIRED ACCURACY	<u>(j)</u>						

- (a) Change in latitude - deg
- (b) Change in longitude - deg
- (c) Change in altitude - km
- (d) Change in velocity - km/sec
- (e) Change in azimuth - deg
- (f) Change in flight path angle - deg
- (g) Change in initial time - hrs
- (h) Code indicating quantities (up to 7) to be searched for
- (i) Desired values of above quantities
- (j) Tolerances allowed on above values

MATRIX OF PARTIAL DERIVATIVES	<u>(k)</u>		
RESIDUALS AND CHANGES IN INITIAL CONDITIONS	<u>(l)</u>	<u>(m)</u>	<u>(n)</u>

- (k) Matrix having the dependent variables arranged by row; the independent by column
- (l) Residuals (desired-nominal) of quantities designated by the quantity code
- (m) Changes required in initial conditions
- (n) Normalized changes in initial quantities in order of the variations

#### 14. Impact Parameter Output

IMPACT PLANE	PARAMETERS	DIRECTION	COSINES OF
T VECTOR	<u>(a)</u>		
R VECTOR	<u>(b)</u>		

ASYMPTOTE	<u>(c)</u>
MISS PARAMETERS CROSS T	<u>(d)</u>
B · T , B · R	<u>(e)</u> <u>(f)</u>

- (a) Unit vector parallel to the ecliptic or moon orbital plane and perpendicular to the incoming asymptote
- (b) Unit vector perpendicular to T-vector and the asymptote in a right-hand sense
- (c) Unit vector in the direction of the incoming asymptote
- (d) Time at which the impact plane is crossed - hrs
- (e) The dot product of  $R_{VT}$  at crossing time and T-vector - km
- (f) The dot product of  $R_{VT}$  at crossing time and R-vector - km

(a), (b) and (c) are printed after the set up of the impact plane coordinate system. This occurs when the nominal trajectory reaches the required approach distance. This will not be repeated for subsequent trajectories unless NOMTR is reset to zero.

(d), (e) and (f) are printed after the spacecraft crosses the impact plane.

## 15. Overflow Diagnostic

ADDRESS PART OF 3 PRECEDING INSTRUCTIONS  
AND IR STATUS

(a)

(b)

NUMBERS IN LISTED LOCATIONS,  
AC, MQ

(c) (d) (e)

- (a) Address part of the three instructions preceding the location of the overflow
- (b) The contents of index registers 1, 2, 4 at the time of the overflow
- (c) The three numbers in the locations referred to in (a)
- (d) The contents of the accumulator
- (e) The contents of the multiplier quotient register

16.. Earth-Fixed Velocity

EARTH-FIXED VELOCITY X = (a) Y = (b) Z = (c)

V = (d) AZ = (e) EL = (f)

- (a) x-component of the earth-fixed velocity - km/sec
- (b) y-component of the earth-fixed velocity - km/sec
- (c) z-component of the earth-fixed velocity - km/sec
- (d) Magnitude of earth-fixed velocity - km/sec
- (e) Geocentric flight path azimuth - deg
- (f) Geocentric flight path angle - deg

## XI. INTERNAL PROCEDURES

### A. Units

The units which are used internally are earth radii and earth radii per hour in earth and moon references, and astronomical units and astronomical units per hour in sun and planet reference systems.

### B. Ephemeris Tape

1. The relative positions of the solar system bodies are obtained from cards furnished by the U. S. Naval Observatory. A separate program prepares a binary tape referred to the mean equinox of date with 15 days per record in a form compatible with the main program. The main program searches the tape and reads in the proper file and record, keeping 30 days of tables in core storage at a time.

The first record on each file consists of the year in fixed decimal. Each of the following records contain the following information:

Word 1: Initial time of record in hours from base time.

Then 12 consecutive 15 word blocks (one word per day) containing the equatorial coordinates of:

XSE	YSE	ZSE	Sun with respect to earth
XJS	YJS	ZJS	Jupiter with respect to the sun
XAS	YAS	ZAS	Mars with respect to the sun
XVS	YVS	ZVS	Venus with respect to the sun

Then three 30 word blocks containing:

XME	YME	ZME	Moon with respect to earth
-----	-----	-----	----------------------------

The moon coordinates are stored in half-day intervals ( $0^h.0$ ,  $12^h.0$  UT). The unit of distance is the radius of the earth. All other tables are in daily intervals ( $0^h.0$  UT) and the unit of distance is AU.

At present, an ephemeris tape is available for 1961-1970, written in nine, two-year files. The files overlap one year.

2. The tape generated from JPL data is written in two-year overlapping files covering the period 1968-1982. The first record of each file has the starting year twice, then the number of records and, finally, the number of files. Each succeeding record contains the following:

Time in hours with respect to the beginning of the file.

Mercury coordinates with respect to the sun.

9 x-values	4 days apart
9 y-values	4 days apart
9 z-values	4 days apart

Venus coordinates with respect to the sun.

5 x-values	8 days apart
5 y-values	8 days apart
5 z-values	8 days apart

These are followed by the coordinates of the sun with respect to the earth, Mars with respect to the sun, Jupiter with respect to the sun, Saturn with respect to the sun, Uranus with respect to the sun, Neptune with respect to the sun, Pluto with respect to the sun, and earth-moon barycenter with respect to the sun, all eight days apart.

The positions of the moon with respect to the earth follow:

33 x-values	half-day intervals
33 y-values	half-day intervals
33 z-values	half-day intervals

These position values are then followed by their respective velocities in the same order.

### C. Ephemeris in Core

1. The astronomical tables are stored in core using 24-hour intervals for the sun and planets, and 12-hour intervals for the moon. There are always 30 days of tables available, arranged in such a way that the value of time for which the interpolation takes place is not near either end of the table.

In location TABLE is stored the time of the first entry from the initial time. Stored in the following locations are:

TABLE + 1 to TABLE + 30                    x coordinates of the sun

TABLE + 31 to TABLE + 60	y coordinates of the sun
TABLE + 61 to TABLE + 90	z coordinates of the sun
TABLE + 91 to TABLE + 180	x,y,z coordinates of Jupiter
TABLE + 181 to TABLE + 270	x,y,z coordinates of Mars
TABLE + 271 to TABLE + 360	x,y,z coordinates of Venus
TABLE + 361 to TABLE + 420	x coordinates of the moon
TABLE + 421 to TABLE + 480	y coordinates of the moon
TABLE + 481 to TABLE + 540	z coordinates of the moon

2. Using the JPL ephemeris tape, the astronomical tables stored in core are:

TABLE	time
TABLE + 1 to TABLE + 9	XSE
TABLE + 10 to TABLE + 18	YSE
TABLE + 19 to TABLE + 27	ZSE
TABLE + 28 to TABLE + 36	XJPS
TABLE + 37 to TABLE + 45	YJPS
TABLE + 46 to TABLE + 54	ZJPS
TABLE + 55 to TABLE + 63	XMRS
TABLE + 64 to TABLE + 72	YMRS
TABLE + 73 to TABLE + 81	ZMRS
TABLE + 82 to TABLE + 90	XVNS
TABLE + 91 to TABLE + 99	YVNS
TABLE + 100 to TABLE + 108	ZVNS
TABLE + 109 to TABLE + 173	XME
TABLE + 174 to TABLE + 238	YME
TABLE + 239 to TABLE + 303	ZME

The velocities are stored in the same manner as follows:

TABLE + 304 to TABLE + 312	XDSE
TABLE + 313 to TABLE + 321	YDSE
TABLE + 322 to TABLE + 330	ZDSE
TABLE + 331 to TABLE + 339	XDJPS
TABLE + 340 to TABLE + 348	YDJPS
TABLE + 349 to TABLE + 357	ZDJPS
TABLE + 358 to TABLE + 366	XDMRS
TABLE + 367 to TABLE + 375	YDMRS
TABLE + 376 to TABLE + 384	ZDMRS
TABLE + 385 to TABLE + 393	XDVNS
TABLE + 394 to TABLE + 402	YDVNS
TABLE + 403 to TABLE + 411	ZDVNS
TABLE + 412 to TABLE + 476	XDME
TABLE + 477 to TABLE + 541	YDME
TABLE + 542 to TABLE + 606	ZDME

#### D. Perturbation Program

The perturbation program solves three differential equations for XI, ETA, ZETA. The differential equation for XI, with the various terms replaced by the storages containing them, is representative of all three equations and is given here.

```
D2XI = - GME  (VCOR + 0/VCOR + 3 - COMP + 0/COMP + 3)
      - GMVN (VCOR + 18/VCOR + 21 - COMP + 18/COMP + 21)
      - GMS  (VCOR + 12/VCOR + 15 - COMP + 12/COMP + 15)
      - GMMR (VCOR + 24/VCOR + 27 - COMP + 24/COMP + 27)
      - GMJP (VCOR + 30/VCOR + 33 - COMP + 30/COMP + 33)
      - GMM  (VCOR + 6/VCOR + 9 - COMP + 6/COMP + 9)
      + OTHER PERTURBATIONS
```

where, e.g., in the first term  $GME = K^2$  times the mass of the earth, and  $VCOR + 3$  is the length cubed of the vector  $(VCOR + 0, VCOR + 1, VCOR + 2)$ . Similarly, in the other terms the denominator is the length cubed of the corresponding numerator. In case the two terms within each parenthesis are nearly equal, they are computed by the special method described in Appendix E to avoid loss of accuracy. The contents of the COMP storage at any time,  $t$ , depends upon the reference origin at that time. For details see Tables 3 and 4.

Here  $XME$  refers to the  $x$ -component of the vector from earth to moon, with corresponding definitions for the other quantities. An additional subscript of 0 to any quantity denotes the vector derived from the two-body problem.

TABLE 3 - CONTENTS OF COMP TO COMP + 33

Earth Ref	Moon Ref	Sun Ref	Venus Ref	Mars Ref	Jupiter Ref	Storage
XVE0	XME	XSE	XVNE	XMRE	XJPE	COMP + 0, + 1, + 2, 3, 4, 5
XEM	XVM0	XSM	XVNM	XMRM	XJPM	COMP + 6, + 7, + 8
XES	XMS	XVS0	XVNS	XMRS	XJPS	COMP + 12, + 13, + 14
XEVN	XMVN	XSVN	XVVN0	XMRVN	XJPVN	COMP + 18, + 19, + 20
XEMR	XMMR	XSMR	XVNMR	XVMR0	XJPMR	COMP + 24, + 25, + 26
XEJP	XMJP	XSJJP	XVNJP	XMRJP	XVJP0	COMP + 30, + 31, + 32

TABLE 4 - CONTENTS OF VCOR TO VCOR + 33

All References	Storage
XVE	VCOR + 0, + 1, + 2
XVM	VCOR + 6, + 7, + 8
XVS	VCOR + 12, + 13, + 14
XVVN	VCOR + 18, + 19, + 20
XVMR	VCOR + 24, + 25, + 26
XVJP	VCOR + 30, + 31, + 32

## XII. OPERATING PROCEDURES

### A. Introduction

The following procedures for running this program are assembled to serve as a guide for programmers and IBM 7094 machine operators when using the Interplanetary Trajectory Encke Method program. It is strongly advised that, before using the program, all input should be listed and examined carefully, then fastened to output when received in order to provide a permanent record of input for each case run.

### B. Normal Operation (Non-Monitor)

1. Input: The data are supplied on cards. They should be loaded on tape and mounted on A2. The last card of the input data should be a TRA 37777 (for non-monitor operation) beginning in column 8. The normal program stop in this case will show location 37777 in the instruction counter and HTR 37777 in storage.
2. Output: The output of the program is on tape A3.
3. Ephemeris Tapes: A variety of ephemeris tapes (tables of planetary positions as functions of time and year) may be used. Normally a 10-year tape will be used. The particular tape desired will be noted from the input instructions by tape number. These tapes may be either high or low density and should be so marked on the tape reel along with the other identifying information. The ephemeris tape should be mounted on A5 with the appropriate density. These tapes should all be FILE PROTECTED.

### C. Monitor Operation

For monitor operation, the input card TRA 37777 is omitted and the program followed by the input is loaded off-line on A2. The end of file mark on A2 after all cards are read will then return control to the monitor. The loader (first card) must be replaced by an A2 loader.

The instructions on the call card are as follows:

00000	0	00025	0	00003	IOCD	
00001	0	06000	0	00001	TCOA	
00002	0	76000	0	00004	ENK	
00003	0	13100	0	00000	XCA	
00004	0	73400	1	00000	PAX	
00005	0	77200	0	02205	REWB	5
00006	-2	00001	1	00014	TNX	14, 1, 1
00007	0	76200	0	02225	RTBB	5
00010	-0	54000	0	00022	RCHB	
00011	0	06100	0	00011	TCOB	
00012	2	00001	1	00007	TIX	
00013	-0	03000	0	00014	TEFB	
00014	-0	02200	0	00015	TRCB	
00015	0	76200	0	02225	RTBB	5
00016	-0	54000	0	00021	RCHB	
00017	-0	54400	0	00000	LCHB	
00020	0	02000	0	00001	TRA	
00021	-1	00003	0	00000	IOCT	
00022	2	00000	0	00000		
00023	1	00000	0	00022		

#### Operating Procedure

- a. Place input tape on A2
- b. Place output tape on A3
- c. Place ephemeris tape on A5
- d. Place program tape on B5
- e. All sense switches up
- f. Key number 35 down (for first file on B5)
- g. Clear machine
- h. Ready B5 call card in card reader
- i. Press load cards key

Program will start and continue to normal stop; IC 37777 and HTR 37777 in storage.

D. Alternate Method of Operation

1. Occasionally it is useful to place the data immediately behind the binary deck and load the program directly into the machine from tape A2. The A2 loader must replace the B5 loader as the first card in the deck and the A2 call card is used to initiate the program.
2. To run with deck loaded on A2
  - a. Place A2 loader on binary deck
  - b. Place data behind binary deck
  - c. Load cards onto tape; mount on A2
  - d. Place planetary ephemeris tape on A5
  - e. Key 35 down
  - f. Place A2 call card in card reader
  - g. Press load cards key
  - h. Output is on A3

E. Error Stops

Numerous error stops are provided when the machine detects inconsistencies. In this case, the machine prints ERROR AT LOCATION ..... , then prints a number of working storages to aid in finding the cause of the error and proceeds to the next case. Most frequently, such conditions are caused by faulty input. As a first step, therefore, the input should be examined carefully. In this procedure it is extremely helpful to list the input before executing the program and clipping it to the output.

F. Machine Requirements

The program uses a 32K, 2-channel IBM 7090 or 7094 computer. It uses 3 tapes and 1 printer on A channel and 1 tape on B channel. A second tape is used on B channel if the save-restore feature is used. For monitor operation, or the alternate mode described in Section XII-D, the B channel is unnecessary.

G. Additional Information

For additional information to aid in operating this program, please contact Mr. F. H. Whitlock, Goddard Space Flight Center, Theoretical Division, Greenbelt, Maryland 20771.

### XIII. LIST OF IMPORTANT LOCATIONS IN THE PROGRAM

This section presents a list of locations of constants and other quantities which will enable the program user to modify the computations for special purposes.

The majority of the solar system constants, e.g., earth gravitational harmonics, planetary masses, etc., are found in the program listing beginning at 25415. The first group of coefficients are used in the series expansion for the sub-satellite computation. Notable exceptions are as follows:

#### A. In Radar

24406	Z\$ESQ	Earth's eccentricity squared
24407	Z1MESQ	One minus earth's eccentricity squared
24411	Z\$F	Reciprocal of earth's radius - feet

#### B. In Drag Table

27763	DRSC	Negative earth radius - cm. Drag scalar
-------	------	---

#### C. Other Locations

30546	XRDT	Velocity of vehicle with respect to reference body
30535	XRODT	Unperturbed velocity of vehicle with respect to reference body
30363	T	Time from launch - hrs
30364	$h = T + 1$	Current integration interval - hrs
30365 30367	} XI	Perturbation displacement
30370 30372	} XI+3	Perturbation velocity
30373 30375	} D2XI	Perturbation Acceleration
50472	TABLE	Beginning of a block of locations described in Section XI

#### XIV. REFERENCES

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## MATHEMATICAL APPENDIX

### A. INTRODUCTION

The problem of orbit determination over long time periods requires a precise technique for integrating the equations of motion. Reference 4 contains an analysis of integration procedure that yields the minimum loss of information due to the accumulation of numerical round-off errors. The Encke perturbation method has been shown to require minimum machine computation time for a minimum loss of numerical accuracy. The orbit prediction scheme presented herein uses a modified form of the Encke method with the initial position and velocity vectors replacing the conventional P and Q vectors of the Encke scheme. By avoiding reference to the position of perigee, it is possible to avoid numerical ambiguities arising from near circular orbits and orbits of low inclination.

## B. EQUATIONS OF MOTION

In a Newtonian system, the equations of motion of a particle in the gravitational field of  $n$  attracting bodies and subject to other perturbing accelerations such as thrust, drag, oblateness, radiation pressure, etc. are given by

$$\ddot{\mathbf{R}}_v = - \sum_{i=1}^n \mu_i \frac{\mathbf{R}_{vi}}{r_{vi}^3} + \sum_j \mathbf{F}_j \quad (B.1)$$

These equations are put into observable form by referring them to a reference body  $c$ . The equations of motion of the reference body are

$$\ddot{\mathbf{R}}_c = - \sum_{\substack{i=1 \\ i \neq c}}^n \mu_i \frac{\mathbf{R}_{ci}}{r_{ci}^3} \quad (B.2)$$

Subtraction of Equation (B.2) from Equation (B.1) results in the equation of motion of the vehicle with respect to the reference body  $c$ .

$$\ddot{\mathbf{R}}_{vc} = - (\mu_v + \mu_c) \frac{\mathbf{R}_{vc}}{r_{vc}^3} - \sum_{\substack{i=1 \\ i \neq c}}^n \mu_i \left[ \frac{\mathbf{R}_{vi}}{r_{vi}^3} - \frac{\mathbf{R}_{ci}}{r_{ci}^3} \right] + \sum_j \mathbf{F}_j \quad (B.3)$$

C. METHOD OF INTEGRATION

If Equation (B. 3) is integrated directly by some numerical scheme, there results, after a number of step-by-step integrations, an accumulation of error which leads to inaccurate results. To avoid this loss in precision, it is convenient to write Equation (B. 3) in the form

$$\ddot{\mathbf{R}}_{vc} = \ddot{\mathbf{R}}_k + \Delta \ddot{\mathbf{R}} \quad (C. 1)$$

The velocity and displacement vectors can be written as

$$\dot{\mathbf{R}}_{vc} = \dot{\mathbf{R}}_k + \Delta \dot{\mathbf{R}} \quad (C. 2)$$

$$\mathbf{R}_{vc} = \mathbf{R}_k + \Delta \mathbf{R} \quad (C. 3)$$

The reference body (the one in whose sphere of influence the vehicle travels) is chosen so as to minimize the perturbations.

In this method  $\ddot{\mathbf{R}}_k$  is taken as

$$\ddot{\mathbf{R}}_k = -(\mu_v + \mu_c) \frac{\mathbf{R}_k}{r_k^3} \quad (C. 4)$$

and

$$\ddot{\xi} = -(\mu_v + \mu_c) \left[ \frac{\mathbf{R}_{vc}}{r_{vc}^3} - \frac{\mathbf{R}_k}{r_k^3} \right] - \sum_{\substack{i=1 \\ i \neq c}}^n \mu_i \left[ \frac{\mathbf{R}_{vi}}{r_{vi}^3} - \frac{\mathbf{R}_{ci}}{r_{ci}^3} \right] + \sum_j \mathbf{F}_j \quad (C. 5)$$

Equations (C. 4) constitute the equations of motion of the Kepler problem and are solved as described in Appendix D.

Equations (C.5) are integrated numerically. The integration scheme employed by the ITEM program is a sixth order backward difference scheme, initiated by a Runge-Kutta scheme. The routine used is "RW DE6F Floating Point Cowell Second Sum Integration of Second Order Differential Equations." (SHARE distribution #775.)

As derived in the following subsection, the solution of the Kepler problem may be represented by the vectors  $\mathbf{R}_o, \dot{\mathbf{R}}_o$ , the scalar  $a$ , and the rectification time  $t_o$ .

The rectification process consists of moving  $\mathbf{R}_{vc}, \dot{\mathbf{R}}_{vc}$  into the locations  $\mathbf{R}_o$  and  $\dot{\mathbf{R}}_o$ ,  $t$  into  $t_o$  and computing  $a$  and  $n$ . For computational convenience, the coefficients appearing in Equations (D.2) are also computed during rectification.

D. SOLUTION OF THE KEPLER TWO-BODY PROBLEM

The unified formulation of the two-body problem is used for both elliptic and hyperbolic cases.

$$\beta = \sqrt{|a|} \cdot \theta$$

$$\alpha = \beta^2 \left( \frac{1}{a} \right)$$

$$F_1(\alpha) = \frac{1}{6} - \frac{\alpha}{120} + \frac{\alpha^2}{5040} \dots = \sum_{i=0}^{\infty} \frac{(-\alpha)^i}{(2i+3)!} \quad (D.1)$$

$$F_2(\alpha) = \frac{1}{2} - \frac{\alpha}{24} + \frac{\alpha^2}{720} \dots = \sum_{i=0}^{\infty} \frac{(-\alpha)^i}{(2i+2)!}$$

$$F_3(\alpha) = 1 - \alpha F_1$$

$$F_4(\alpha) = 1 - \alpha F_2$$

and

$$f = 1 - \frac{1}{r_0} \beta^2 F_2$$

$$g = \frac{r_0}{\sqrt{\mu}} \beta F_3 + \frac{d_0 \beta^2 F_2}{\mu} \quad (D.2)$$

$$\dot{f} = -\frac{\sqrt{\mu}}{r_0 r} \beta F_3$$

$$\dot{g} = 1 - \frac{1}{r} \beta^2 F_2$$

where

$$d_o = R_o \cdot \dot{R}_o$$

$$r = \beta^2 F_2 + r_o F_4 + \frac{d_o}{\sqrt{\mu}} \beta F_3$$

$$R = f R_o + g \dot{R}_o$$

$$\dot{R} = f \dot{R}_o + g \ddot{R}_o$$

$$a = \left( \frac{2}{r_o} - \frac{v_o^2}{\mu} \right)^{-1}$$

$$v_o^2 = \dot{R}_o \cdot \ddot{R}_o$$

$\alpha$  is determined from the modified Kepler equation

$$\sqrt{\mu} \Delta t = \beta^3 F_1 + r_o \beta F_3 + \frac{d_o}{\sqrt{\mu}} \beta^2 F_2 \quad (D.3)$$

See Figure 4 for the two-body orbit which results from the solution of Equation (C.4) with the initial conditions:

$$\begin{aligned} R_k(t_o) &= R_{vc}(t_o) = R_o \\ \dot{R}_k(t_o) &= \dot{R}_{vc}(t_o) = \dot{R}_o \end{aligned} \quad (D.4)$$

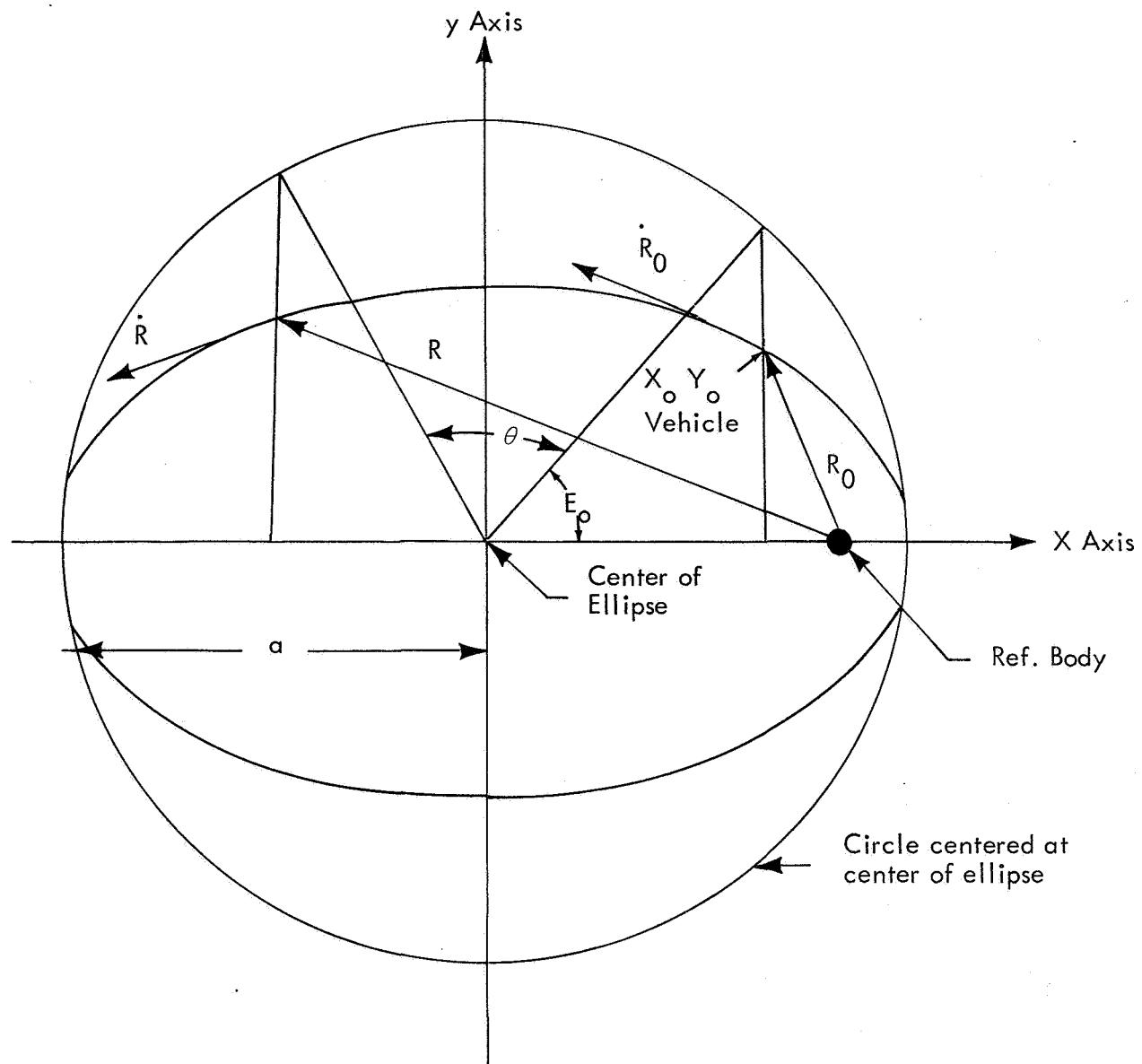


Figure 4. The Geometry of the Elliptic Kepler Orbit.

E. COMPUTATION OF PERTURBATION TERMS

The terms accounting for the Encke term and the planetary perturbations appearing on the right hand side of Equation (C. 5) involve

numerous terms of the form  $\frac{R}{r^3} - \frac{R_o}{r_o^3}$  where  $R$  and  $R_o$  may differ

only by small amounts. For the Encke term, for instance  $R - R_o = \xi$  which is small, and for the planetary perturbations, the difference is  $R_{vc}$  which also often is small.

A computation scheme, which avoids loss of precision due to the subtraction of nearly equal terms and which also is correct when  $R_{vc}$  is not small, is employed. This scheme is described below: Find

$$\frac{R}{r^3} - \frac{R_o}{r_o^3}$$

$$u = \frac{2}{r^2} (R_o + \frac{1}{2} \Delta R) \cdot \Delta R \quad (E. 1)$$

$$\frac{R}{r^3} - \frac{R_o}{r_o^3} = \frac{\Delta R}{r_o^3} + \frac{R(u^3 + 3u^2 + 3u)}{\left(1 + \frac{r^3}{r_o^3}\right)}$$

F. CONCLUSIONS

The method presented yields accurate trajectories using relatively little computer time. Summarizing some of the important features:

1. All significant solar system bodies may be included without undue complications.
2. Since the perturbations only are integrated, the allowable integration interval is fairly large over most of the path. Even in the vicinity of earth or another planet a relatively large interval (compared to other schemes) may be used without limiting the stability and accuracy of the solutions.
3. The perturbations are kept small in two ways. First, the two-body orbit is rectified whenever the perturbations exceed a specified maximum value compared to the corresponding unperturbed values. This limits error build-up with respect to a particular reference body. Second, the reference body of the two-body problem is changed from earth, to sun, to planet accordingly, as that reference body would contribute the largest perturbing force otherwise.
4. This method will handle circular orbits, zero inclination, etc. The problem is defined in terms of parameters which have real physical significance (namely, the position and velocity vectors) which are directly relatable to measurable quantities.

G. OBLATENESS TERMS

The oblateness perturbation terms in Equations (C. 5) are derived from the potential given by the following equation:

$$\begin{aligned}\varphi = \frac{\mu}{r} \left\{ - \left( \frac{a_e}{r} \right)^2 J_{20} \left[ \frac{3}{2} \left( \frac{z}{r} \right)^2 - \frac{1}{2} \right] \right. \\ \left. - \left( \frac{a_e}{r} \right)^3 J_{30} \left[ \frac{5}{2} \left( \frac{z}{r} \right)^3 - \frac{3}{2} \left( \frac{z}{r} \right) \right] \right. \\ \left. - \left( \frac{a_e}{r} \right)^4 J_{40} \left[ \frac{35}{8} \left( \frac{z}{r} \right)^4 - \frac{15}{4} \left( \frac{z}{r} \right)^2 - \frac{3}{8} \right] \right\} \quad (G. 1)\end{aligned}$$

where  $a_e$  is the equatorial radius of the earth.

$$\mathbf{F} = \text{grad } \varphi$$

This vector can be written in the form

$$\mathbf{F} = \lambda \mathbf{R} + \mathbf{m} \hat{\mathbf{k}}$$

where  $\hat{\mathbf{k}}$  is a unit vector in the  $z$ -direction.

$$\begin{aligned}\lambda = \frac{\mu}{r^3} \left\{ \left( \frac{a_e}{r} \right)^2 J_{20} \left[ \frac{15}{2} \left( \frac{z}{r} \right)^2 - \frac{3}{2} \right] \right. \\ \left. + \left( \frac{a_e}{r} \right)^3 J_{30} \left[ \frac{35}{2} \left( \frac{z}{r} \right)^3 - \frac{15}{2} \left( \frac{z}{r} \right) \right] \right. \\ \left. + \left( \frac{a_e}{r} \right)^4 J_{40} \left[ \frac{315}{8} \left( \frac{z}{r} \right)^4 - \frac{105}{4} \left( \frac{z}{r} \right)^2 - \frac{15}{8} \right] \right\} \quad (G. 2)\end{aligned}$$

$$\begin{aligned}
 m = & -\frac{\mu}{r^2} \left\{ \left( \frac{a_e}{r} \right)^2 J_{20} \left( 3 \frac{z}{r} \right) + \left( \frac{a_e}{r} \right)^3 J_{30} \left[ \frac{15}{2} \left( \frac{z}{r} \right)^2 - \frac{3}{2} \right] \right. \\
 & \left. + \left( \frac{a_e}{r} \right)^4 J_{40} \left[ \frac{35}{2} \left( \frac{z}{r} \right)^3 - \frac{15}{2} \left( \frac{z}{r} \right) \right] \right\}
 \end{aligned} \tag{G. 3}$$

The perturbation acceleration due to tesseral harmonics are computed.

$$\text{Constants: } J_2^2, J_3^1, J_3^3, \lambda_2^2, \lambda_3^1, \lambda_3^3$$

$$\begin{aligned}
 J_2^2 &= -1.9E-6 & \lambda_2^2 &= -21.^\circ \\
 J_3^1 &= -1.51E-6 & \lambda_3^1 &= 0 \\
 J_3^3 &= -1.149E-6 & \lambda_3^3 &= 22.8^\circ
 \end{aligned} \tag{G. 4}$$

In initialization:

$$\begin{aligned}
 C_i^j &= J_i^j \cos j \lambda_i^j \\
 S_i^j &= J_i^j \sin j \lambda_i^j
 \end{aligned} \tag{G. 5}$$

$x, y, z$  are  $x', y', z'$

### $F_{22}$ Term

$$\begin{aligned}
 A_{22} &= \frac{5}{r^2} (x^2 - y^2) \\
 B_{22} &= -5 \frac{xy}{r^2}
 \end{aligned} \tag{G. 6}$$

$$\begin{aligned}
 F_{22x'} &= \frac{3\mu}{5} \left\{ C_2^2 (2x - xA_{22}) + 2S_2^2 (y - xB_{22}) \right\} \\
 F_{22y'} &= \frac{3\mu}{5} \left\{ C_2^2 (-2y - yA_{22}) + 2S_2^2 (x - yB_{22}) \right\} \\
 F_{22z'} &= \frac{3\mu}{5} \left\{ C_2^2 (-zA_{22}) + 2S_2^2 (-zB_{22}) \right\}
 \end{aligned} \tag{G. 7}$$

$x, y, z$  are earth fixed coordinates.  $x, y$  in the equatorial plane, the  $x$ -axis toward the Greenwich meridian. Nutation is neglected.

$F_{31}$  Term

$$\begin{aligned}
 K_{31} &= (4z^2 - x^2 - y^2) \\
 A_{31} &= \frac{7x}{r^2} K_{31} \\
 B_{31} &= \frac{7y}{r^2} K_{31}
 \end{aligned} \tag{G. 8}$$

$$\begin{aligned}
 F_{31x'} &= \frac{3}{2} \frac{\mu}{r^7} \left\{ C_3^1 [K_{31} - 2x^2 - xA_{31}] + S_3^1 [-2xy - xB_{31}] \right\} \\
 F_{31y'} &= \frac{3}{2} \frac{\mu}{r^7} \left\{ C_3^1 [-2xy - yA_{31}] + S_3^1 [K_{31} - 2y^2 - yB_{31}] \right\} \\
 F_{31z'} &= \frac{3}{2} \frac{\mu}{r^7} \left\{ C_3^1 [8xz - zA_{31}] + S_3^1 [8yz - zB_{31}] \right\}
 \end{aligned} \tag{G. 9}$$

F<sub>33</sub> Term

$$A_{33} = \frac{7x}{r^2} (x^2 - 3y^2) \quad (G. 10)$$

$$B_{33} = \frac{7y}{r^2} (3x^2 - y^2)$$

$$F_{33x'} = \frac{15\mu}{r^7} \left\{ C_3^3 [ 3(x^2 - y^2) - x A_{33} ] + S_3^3 [ 6xy - x B_{33} ] \right\}$$

$$F_{33y'} = \frac{15\mu}{r^7} \left\{ C_3^3 [ -6xy - y A_{33} ] + S_3^3 [ 3(x^2 - y^2) - y B_{33} ] \right\} \quad (G. 11)$$

$$F_{33z'} = \frac{15\mu}{r^7} \left\{ C_3^3 [ -z A_{33} ] - S_3^3 [ z B_{33} ] \right\}$$

$$F_x' = F_{22x'} + F_{31x'} + F_{33x'}$$

$$F_y' = F_{22y'} + F_{31y'} + F_{33y'} \quad (G. 12)$$

$$F_z' = F_{22z'} + F_{31z'} + F_{33z'}$$

H. TRANSFORMATION EQUATIONS FROM GEODETIC POLAR COORDINATES TO CARTESIAN COORDINATES\*

The geodetic polar coordinates in the program are referred to an ellipsoid of revolution. The equation of a cross section is given by

$$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1 \quad (H.1)$$

where

$$b^2 = a^2(1 - e^2)$$

The slope of the normal, along which  $h$  is measured is given by

$$\tan \phi = -\frac{1}{\frac{dz}{dx}} = \frac{a^2 z}{b^2 x} \quad (\text{See Figure 5}) \quad (H.2)$$

and

$$\tan \phi' = \frac{z}{x} = \frac{b^2}{a^2} \tan \phi = (1 - e^2) \tan \phi$$

Eliminating  $x$  between equations (H.1) and (H.2) and solving for  $z$  results in:

$$z = \frac{a(1 - e^2) \sin \phi}{(1 - e^2 \sin^2 \phi)^{1/2}}$$

---

\*For geocentric (i.e.  $e^2 = 0$ ) polar coordinates,  $c = s = 1$ . In this case the latitude input is interpreted as declination.

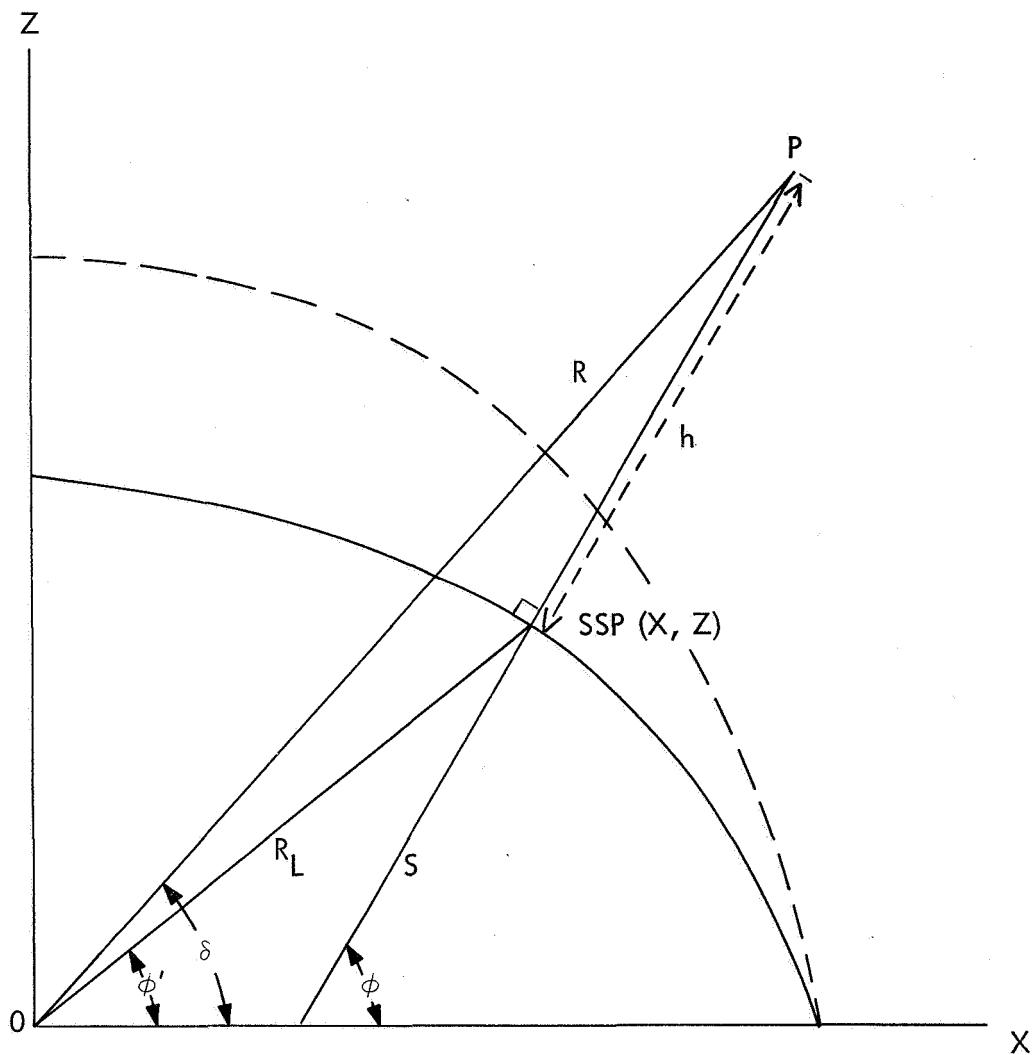


Figure 5. Relation Between Declination, Geocentric  
and Geodetic Latitudes

and from equation (H.2) then

$$x = \frac{a \cos \phi}{(1 - e^2 \sin^2 \phi)^{1/2}}$$

In units of  $a_e$ , R and R are then given by equation (H.3)

$$c = (1 - e^2 \sin^2 \phi)^{-1/2}$$

$$s = (1 - e^2) c$$

$$x = (c + h) \cos \phi \cos (\theta - \theta_0)$$

$$y = (c + h) \cos \phi \sin (\theta - \theta_0)$$

$$z = (s + h) \sin \phi$$

$$\begin{aligned} \dot{x} = v & \left\{ (\sin \gamma \cos \phi - \cos \gamma \cos A \sin \phi) \cos (\theta - \theta_0) \right. \\ & \left. - \cos \gamma \sin A \sin (\theta - \theta_0) \right\} \end{aligned} \quad (H.3)$$

$$\begin{aligned} \dot{y} = v & \left\{ (\sin \gamma \cos \phi - \cos \gamma \cos A \sin \phi) \sin (\theta - \theta_0) \right. \\ & \left. + \cos \gamma \sin A \cos (\theta - \theta_0) \right\} \end{aligned}$$

$$\dot{z} = v \left\{ \sin \gamma \sin \phi + \cos \gamma \cos A \cos \phi \right\}$$

These equations include the effect of the rotation of the earth. The longitude of the vernal equinox ( $\theta_0$ ) at launch time is computed by the program from Newcomb's formula.

## I. TRANSFORMATION EQUATIONS FOR RADAR SIMULATION

The program computes sight angles (in an azimuth-elevation system), slant range and range rate data for up to 30 radar stations. The vehicle coordinates are transformed from a system of geocentric cartesian coordinates (xyz), the x-axis in the direction of the vernal equinox and the x-y plane in the equatorial plane of the earth to the required topocentric azimuth elevation system. This is accomplished by a series of coordinate transformations as follows:

1. A rotation of the coordinate system about the z-axis through an angle  $RA_s$  so that x-y plane is in the meridian plane of the station.

$$x' = x \cos RA_s + y \sin RA_s$$

$$y' = -x \sin RA_s + y \cos RA_s \quad (I.1)$$

$$z' = z$$

The velocity transformation must take the rotational velocity of the new coordinate system into account.

$$\dot{x}' = y' \omega_e + \dot{x} \cos RA_s + \dot{y} \sin RA_s$$

$$\dot{y}' = -x' \omega_e - \dot{x} \sin RA_s + \dot{y} \cos RA_s$$

$$\dot{z}' = \dot{z}$$

where  $x'$ ,  $y'$ ,  $z'$  are the rotated coordinates and  $RA_s$  is the right ascension of the station and  $\omega_e$  is the sidereal rate of the earth's

rotation. The G.H.A. necessary to obtain  $RA_s$  from the station longitude is computed by the program.

2. A translation of the origin of the coordinate system from the center of the earth to the station in question

$$x'' = x' - (c + h) \cos \phi$$

$$y'' = y'$$

$$z'' = z' - (s + h) \sin \phi \quad (I.2)$$

where

$$c = (1 - e^2 \sin^2 \phi)^{-1/2}$$

$$s = (1 - e^2) c$$

$$\dot{x}'' = \dot{x}' ; \quad \dot{y}'' = \dot{y}' ; \quad \dot{z}'' = \dot{z}'$$

where  $x''$ ,  $y''$ ,  $z''$  are the translated coordinates.  $\phi$  is the geodetic latitude and  $h$  the height above sea level of the station in question.

3. A rotation of  $(90 - \phi)$  about the  $y''$  axis to place the  $(x'', z'')$

plane into the horizon plane

$$x''' = x'' \sin \phi + z'' \cos \phi$$

$$y''' = y''$$

$$z''' = -x'' \cos \phi + z'' \sin \phi$$

$$(I.3)$$

$$\dot{x}''' = \dot{x}'' \sin \phi + \dot{z}'' \cos \phi$$

$$\dot{y}''' = \dot{y}''$$

$$\dot{z}''' = -\dot{x}'' \cos \phi + \dot{z}'' \sin \phi$$

Now  $x''$ ,  $y''$ ,  $z''$  are the coordinates of the vehicle in a topocentric azimuth elevation system, with  $z''$  axis pointing to zenith and the  $x''$  pointing south along the meridian. Range, range rate, azimuth and elevation are then given by

$$\rho = \sqrt{x''^2 + y''^2 + z''^2}^{1/2} = \text{Slant range} \quad (I.4)$$

$$\dot{\rho} = \frac{x''' \dot{x}''' + y''' \dot{y}''' + z''' \dot{z}'''}{\rho} = \text{Range rate} \quad (I.5)$$

$$E = \tan^{-1} \frac{z''}{\sqrt{x''^2 + y''^2}} = \text{Elevation} \quad (I.6)$$

$$A' = \tan^{-1} \frac{y''}{x''}$$

$$A = \begin{cases} \pi - A' & A' < \pi \\ 3\pi - A' & A' > \pi \end{cases} \quad (I.7)$$

## J. TRIAXIAL MOON

Triaxial lunar potential constants (as used in the ITEM Program)

1. The values of the constants A, B and C for the perturbation accelerations due to the triaxial moon may be calculated using data from the NASA earth model meeting. These constants are currently being used in the ITEM Program.

2. The perturbation accelerations due to the triaxial moon are given by the partial derivatives of

$$\phi = -\frac{C}{r^3} \left\{ A \left( 1 - \frac{3z^2}{r^2} \right) + B \left( 1 - 3 \frac{x^2}{r^2} \right) \right\} \quad (J.1)$$

where

$$C = \frac{\mu_m a_m^2}{3 a_e^2} \left( \frac{3 I_c}{2 m a_m^2} \right)$$

$$A = \frac{I_c - I_a}{I_c} \quad (J.2)$$

$$B = \frac{I_b - I_a}{I_c}$$

$$\frac{\partial \phi}{\partial x} = \frac{3xC}{r^5} F$$

$$\frac{\partial \phi}{\partial y} = \frac{3yC}{r^5} F - \frac{6CBy}{r^5} \quad (J.3)$$

$$\frac{\partial \phi}{\partial z} = \frac{3zC}{r^5} F - \frac{6ACz}{r^5}$$

where

$$F = \left\{ A \left( \frac{5z^2}{r^2} - 1 \right) + B \left( \frac{5y^2}{r^2} - 1 \right) \right\}$$

Based on the NASA earth model meeting, the moments of inertia about the principal axes of the moon are:

$$I_A = .88746 \times 10^{35} \text{ kg meters}^2$$

$$I_B = .88764 \times 10^{35} \text{ kg meters}^2$$

$$I_C = .88801 \times 10^{35} \text{ kg meters}^2$$

(See figure 6)

Other constants are:

$$\mu_e = 19.9094165 \times \frac{(ER)^3}{m^2}$$

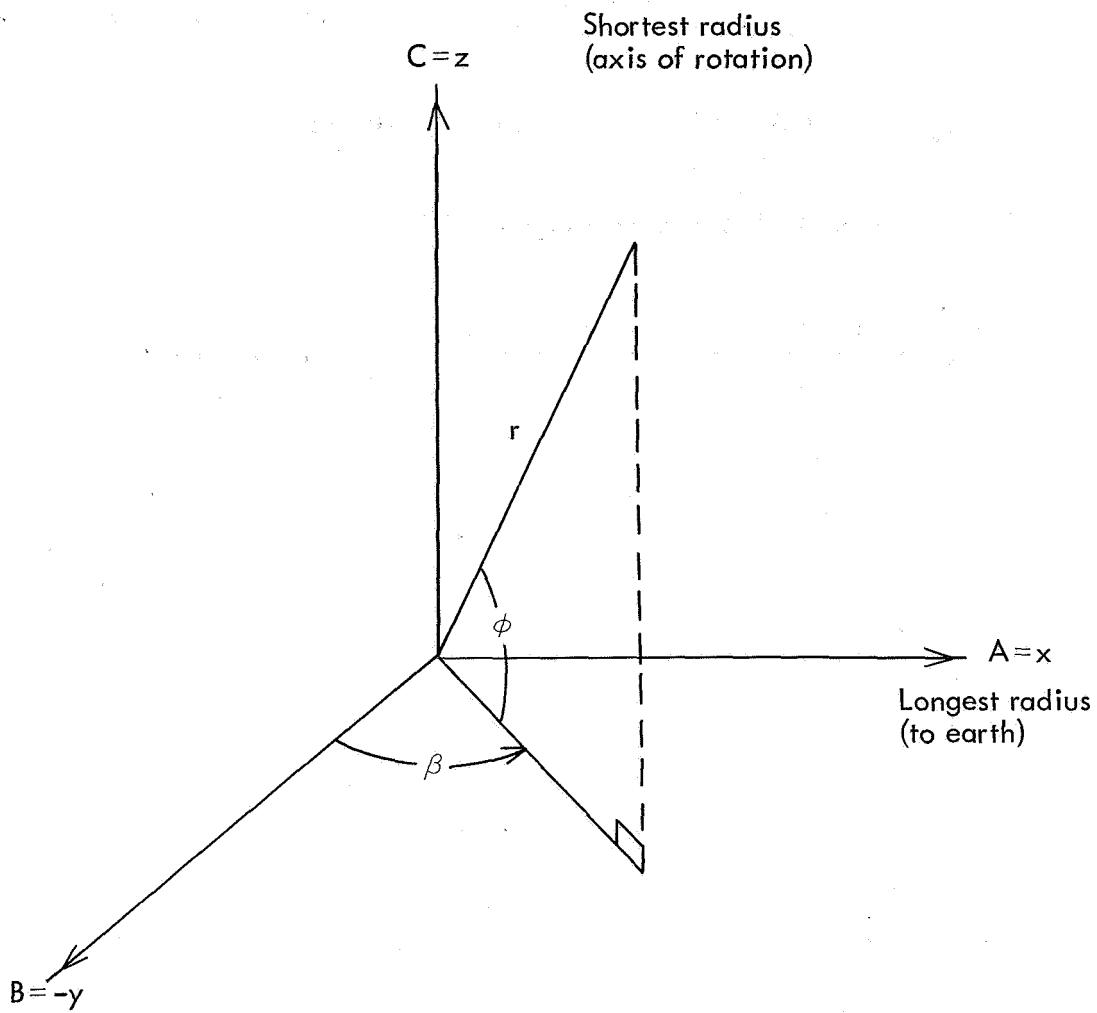


Figure 6. Triaxial Moon

$$\mu_m = \frac{\mu_e}{81.335} = .2447829 \times \frac{(ER)^3}{m^2}$$

$$\frac{(a_m)^2}{(a_e)^2} = \frac{(1738.09)^2}{(6378.165)^2} = .0742595 \text{ (earth radii)}^2$$

$$m_e = 5.975 \times 10^{27} \text{ grams}$$

$$m = \frac{m_e}{81.335} = \frac{5.975 \times 10^{24}}{81.335} = 7.34616 \times 10^{22} \text{ kg}$$

$$a_m^2 = (1738.09)^2 = 3.0209568 \times 10^6 \text{ km}^2$$

The constants A, B and C may be calculated

$$A = \frac{I_c - I_a}{I_c} = \frac{(.88801 - .88746)}{.88801} = 619.36 \times 10^{-6}$$

$$B = \frac{I_b - I_a}{I_c} = \frac{(.88764 - .88746)}{.88801} = 202.70 \times 10^{-6}$$

$$C = \left( \frac{\mu_m a_m^2}{3 a_e^2} \right) \left( \frac{3 I_c}{2m a_m^2} \right)$$

For the ITEM Program the units of C are

$$\frac{(\text{earth radii})^5}{\text{m}^2}$$

therefore

$$C = \left\{ \frac{\mu_m}{3} \left( \frac{a_M}{a_e} \right)^2 \right\} \cdot \left( \frac{3 I_c}{2m a_m^2} \right)$$

$$= \left\{ \frac{(.2447829)(.0742595)}{3} \right\} \left\{ \frac{3(.88801) \times 10^{29}}{2(7.34616 \times 10^{22})(3.0209568 \times 10^8)} \right\}$$

$$C = 36.366998 \times 10^{-4} \text{ earth mass} \times (\text{earth radius})^2$$

In summary the constants used in the ITEM Program as based upon the NASA earth model meeting are:

$$A = 619.36 \times 10^{-6}$$

$$B = 202.70 \times 10^{-6}$$

$$C = 36.366998 \times 10^{-4} \text{ earth mass} \times (\text{earth radius})^2$$

## K. DRAG COMPUTATION

The drag acceleration is computed, assuming a spherically symmetric atmosphere rotating with the earth. Thus:

$$D = -\frac{1}{2} \rho A |V_{eff}| V_{eff}$$
$$\Delta \ddot{R}_D = \frac{D}{m} \quad (K.1)$$

where

$$V_{eff} = \dot{R} - \omega \times R$$

$\omega$  is the sidereal rotation rate vector of the earth.

## L. COMPUTATION OF SUBSATELLITE POINT

The geodetic coordinates of the subsatellite point are computed by the following method:

The geocentric latitude (declination) is obtained from

$$\sin \phi' = \frac{z}{r} \quad (L.1)$$

This latitude is then corrected to geodetic latitude by the formula

$$\phi = \phi' + a_2 \sin 2\phi' + a_4 \sin 4\phi' + a_6 \sin 6\phi' + a_8 \sin 8\phi' \quad (L.2)$$

where

$$a_2 = \frac{1}{1024r} \{512e^2 + 128e^4 + 60e^6 + 35e^8\}$$

$$+ \frac{1}{32r^2} \{e^6 + e^8\} - \frac{3}{256r^3} \{4e^6 + 3e^8\}$$

$$a_4 = - \frac{1}{1024r} \{64e^4 + 48e^6 + 35e^8\}$$

$$+ \frac{1}{16r^2} \{4e^4 + 2e^6 + e^8\} + \frac{15e^8}{256r^3} - \frac{e^8}{16r^4}$$

$$a_6 = \frac{3}{1024r} \{4e^6 + 5e^8\} - \frac{3}{32r^2} \{e^6 + e^8\}$$

$$+ \frac{35}{768r^3} \{4e^6 + 3e^8\} \quad (L.3)$$

$$a_8 = \frac{e^8}{2048} \left\{ -\frac{5}{r} + \frac{64}{r^2} - \frac{252}{r^3} + \frac{320}{r^4} \right\}$$

$e$  = the eccentricity of the earth

$r$  = the distance from earth's center

See Reference 5.

The geodetic height is then given by

$$h = r \cos(\phi - \phi') - \sqrt{1 - e^2 \sin^2 \phi} \quad (L.4)$$

The longitude is obtained by subtracting the sidereal time of Greenwich from the right ascension given by

$$\tan \text{RA} = \frac{y}{x} \quad (L.5)$$

## M. POLAR COORDINATES REFERRED TO THE MOON

Moon longitude and latitude are defined in the coordinate system described in appendix J. If the vehicle coordinates with respect to the center of the moon in this coordinate system are given by  $x$ ,  $y$ ,  $z$ ,  $r$ , then

$$\theta = \text{longitude} = \tan^{-1} \frac{y}{x}$$

$$\phi = \text{latitude} = \sin^{-1} \frac{z}{r}$$

N. SHADOW LOGIC

A coordinate system is set up in the plane defined by the centers of the light-emitting source, the shadowing body, and the probe. Both bodies are assumed to be spherical, and hence all testing can be carried out in this plane. The diagram in Figure 7 shows this plane.

The coordinates are defined by unit vectors  $\underline{i}$  and  $\underline{j}$ :

$$\underline{i} = \frac{\underline{r}_{cl}}{|\underline{r}_{cl}|} ; \quad \underline{i} \cdot \underline{i} = 1 ; \quad \underline{i} \cdot \underline{j} = 0 \quad (N. 1)$$

$$\underline{j} = -d \underline{i} + \underline{R}_{vc} ; \quad \underline{j} \cdot \underline{j} = 1 \quad (N. 2)$$

where

$$d = \underline{R}_{vc} \cdot \underline{i}$$

Vehicle coordinates in this system are given by:

$$x_v = \underline{R}_{vc} \cdot \underline{i} = d \quad (N. 3)$$

$$y_v = \underline{R}_{vc} \cdot \underline{j} = [-d^2 + \underline{r}_{vc}^2]^{1/2} \quad (N. 4)$$

$$z_v = \underline{R}_{vc} \cdot \underline{k} = 0 \quad (N. 5)$$

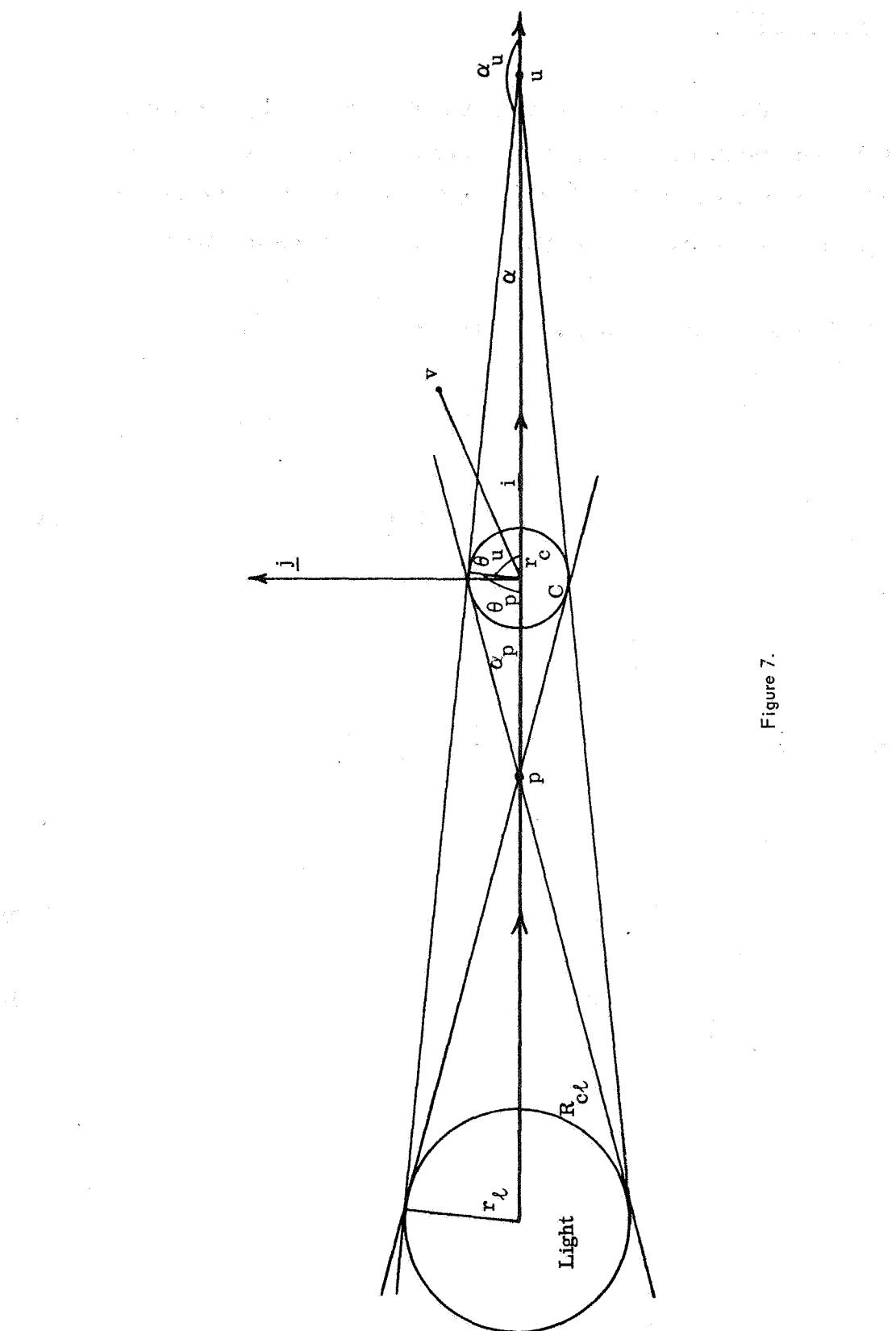


Figure 7.

## 1. Shadow Parameters

a) The tips of the umbra and penumbra cones are:

$$d_u = \frac{r_{cl}}{\frac{r_\ell}{r_c} - 1} \quad , \quad d_p = -\frac{r_{cl}}{\frac{r_\ell}{r_c} + 1}$$

b) The slopes of the bounding lines are:

$\sin \alpha_u = \cos \theta_u = \frac{r_c}{d_u}$ $-\cos \alpha_u = \sin \theta_u = \left[ 1 - \left( \frac{r_c}{d_u} \right)^2 \right]^{\frac{1}{2}}$ $\tan \alpha_u = \frac{\sin \alpha_u}{\cos \alpha_u}$ $\tan \theta_u = \frac{\sin \theta_u}{\cos \theta_u}$	$\cos \theta_p = \sin \alpha_p = \frac{r_c}{d_p}$ $\sin \theta_p = \cos \alpha_p = \left[ 1 - \left( \frac{r_c}{d_p} \right)^2 \right]^{\frac{1}{2}}$ $\tan \alpha_p = \frac{\sin \alpha_p}{\cos \alpha_p}$ $\tan \theta_p = \frac{\sin \theta_p}{\cos \theta_p}$
--	---

c) Refraction Correction: (UMBRA)

$$\alpha'_u = \alpha_u - \epsilon \quad , \quad \theta'_u = \theta_u - \epsilon$$

$$\sin \alpha'_u = \sin \alpha_u \cos \epsilon - \cos \alpha_u \sin \epsilon$$

$$\tan \alpha'_u = \frac{\tan \alpha_u - \tan \epsilon}{1 + \tan \alpha_u \tan \epsilon}$$

$$\tan \theta'_u = \frac{\tan \theta_u - \tan \epsilon}{1 + \tan \theta_u \tan \epsilon}$$

$$d'_u = \frac{r_c}{\sin \alpha'_u}$$

d) Refraction Correction: (PENUMBRA)

$$\text{Both } \epsilon < \alpha_p \quad \epsilon > \alpha_p \quad ; \quad \alpha'_p = \alpha_p - \epsilon$$

$$\sin \alpha'_p = |\sin \alpha_p \cos \epsilon - \cos \alpha_p \sin \epsilon|$$

$$\tan \alpha'_p = \frac{\tan \alpha_p - \tan \epsilon}{1 + \tan \alpha_p \tan \epsilon}$$

$$\tan \theta'_p = \frac{\tan \theta_p - \tan \epsilon}{1 + \tan \theta_p \tan \epsilon}$$

$$d'_p = -\text{sign}(\tan \alpha'_p) \frac{r_c}{\sin \alpha'_p}$$

The equations of the bounding lines are given below.

## 2. The Testing Procedure

$\theta_p$  Line

$$\frac{|y_v|}{\tan \theta'_p} - x_v \geq 0 \quad \text{Sunlight}$$

$$Q_1 = \frac{|y_v|}{\tan \theta'_p} - x_v < 0 \quad \text{Go to next test}$$

$$\begin{aligned}
 Q_2 = & \quad |y_v| - (x_v - d'_p) \tan \alpha'_p > 0 \text{ Sunlight} \\
 & |y_v| - (x_v - d'_p) \tan \alpha'_p = 0 \text{ Sunlight penumbra boundary} \\
 & < 0 \text{ Go to next test}
 \end{aligned}$$

If  $R_\ell = 0$ , exit here.

$$\begin{aligned}
 Q_3 = & \frac{|y_v|}{\tan \theta'_u} - x_v \geq 0 \text{ Penumbra} \\
 & < 0 \text{ Go to next test}
 \end{aligned}$$

$$\begin{aligned}
 & > 0 \text{ Penumbra} \\
 Q_4 = & |y_v| - (x_v - d'_u) \tan \alpha'_u = 0 \text{ Shadow penumbra boundary} \\
 & < 0 \text{ Shadow}
 \end{aligned}$$

$Q_2$  and  $Q_4$  are stored and saved. The crossing times are found by linearly interpolating for 0-values of  $Q_2$  and  $Q_4$  respectively, to guarantee that crossing from one region into another always occurs across these boundaries.

## O. SOLAR RADIATION PRESSURE

The radiation pressure subroutine computes the force of solar radiation on the spacecraft if an appropriate pressure coefficient is used. The calculation relies on the shadow routine to set a trigger to multiply the pressure coefficient by 1.0, 0.5, or 0.0 for full sunlight, penumbra or umbra, respectively. Therefore, the shadow subroutine must be used in conjunction with the radiation pressure routine for most cases. If the spacecraft is known to be continually in sunlight the number 1. may be loaded into SHADIN and thus elaborate shadow testing may be avoided.

$$P_{RP} = \frac{C_R A R_{VS}}{m r_{VS}^3} \quad (O.1)$$

(See Section VIII-H-4. for definition of symbols.)

This radiation pressure subroutine has been found to be inexact for satellites of large area to mass ratio since it only controls the pressure to the nearest integration step. For such spacecraft (e.g., balloons) several degrees error in true anomaly may result after 100 days unless the integration is carried exactly to the boundaries. A modification to achieve this increased precision is available and will be included in future versions of the program.

P. ECLIPTIC COORDINATES

The ecliptic coordinates are an approximate set obtained by a simple rotation of the equatorial coordinates about the x-axis through a fixed angle  $i = 23^\circ 26' 31''$  which is approximately the true obliquity for Jan 0.0, 1962. More exact coordinates may be obtained by changing  $NE$ , (unit normal to the ecliptic) as desired.

Q. MOON ROTATING AND FIXED COORDINATE SYSTEM

Geocentric coordinates of the vehicle based on the earth-moon plane are generated from the geocentric equatorial radius vector to the vehicle,  $R_{VE}$ , the geocentric unit vector in the direction of the moon  $\hat{R}_{ME}$ , and the vector in the direction of the moon's velocity,  $\dot{R}_{ME}$ .

Coordinates in the rotating system, XROT etc., are found by using the current values of these vectors at each time step in the relations.

$$XROT = R_{VE} \cdot \hat{R}_{ME}$$

$$YROT = R_{VE} \cdot (\hat{H}_{ME} \times \dot{\hat{R}}_{ME}) \quad (Q.1)$$

$$ZROT = R_{VE} \cdot \hat{H}_{ME}$$

where

$$\hat{H}_{ME} = \frac{\dot{\hat{R}}_{ME} \times \dot{R}_{ME}}{|\dot{\hat{R}}_{ME} \times \dot{R}_{ME}|}$$

For the fixed axis system XINJ, etc., the initial vectors  $\hat{R}_{ME}(t_0)$  and  $\dot{\hat{R}}_{ME}(t_0)$  at the time of injection are used with the current value of  $R_{VE}$ .

R. TRAJECTORY SEARCH

The program provides a search routine to obtain selected trajectories. The search is based on linear theory and varies the polar load input quantities (independent variables) to search for desired dependent variables. There are twelve possible dependent variables to select from, although a maximum of seven of the twelve may be used in any given search. The quantities, at present, are

$i$ ,  $\Omega$ ,  $\omega$ ,  $t_p$  (pericenter time), and

$r_p$  (pericenter radius)

The above five variables at the moon and at the earth (in the case of earth return trajectories) constitute the first ten variables. (These quantities are normally referred to the equatorial plane. However, the earth-moon plane is also available. See Section VIII.)

In addition, the components of the impact parameter vector ( $B \cdot T$ ,  $B \cdot R$ ) may be selected. They are referred to the ecliptic plane for Mars and Venus trajectories and the moon's orbital plane for lunar trajectories. The number of independent variables need not be equal to the number of dependent variables for this routine to operate. (See Section VIII-C-1.)

This search routine is time consuming if the initial conditions are poorly approximated. Before using this routine, two things should be done.

1. A first guess of the initial conditions of the nominal trajectory should be obtained from a patched conic or a similar search program.
2. The number of variables should be kept to a minimum. It is planned to automate the iteration scheme to go from two-body, to patched conic, to full trajectory, and to increase the number of variables to be adjusted, in optimal fashion. Even in its present form, however, it is extremely useful.

The iterator uses a modified version of the MIN-MAX Principle (Reference 6).

$A_{ij}$  is the matrix of partials

$\Delta x_i$  is the vector of changes in the independent variables

$\lambda_{ii}$  is a diagonal matrix of weights

$y_i$  is a vector of residuals

The system to be solved is

$$(A_{ji} A_{ij} + \lambda_{ii}) \Delta x_i = A_{ji} y_i$$

$$A_{ji} A_{ij} + \lambda_{ii} = B_{ij}$$

$$A_{ji} y_i = z_i$$

### Procedure

The system

$$B_{ij} \Delta x_i = z_i$$

is solved. If the value of  $\Delta x_i$  is greater than SIZER, some arbitrary amount, set

$$B_{ij} = B_{ij} + \lambda_{ii}$$

and solve the system again. Repeat these operations until  $\Delta x_i$  is less than or equal to SIZER. Now, run a new nominal trajectory with the new independent variable

$$x_i = x_i + \Delta x_i$$

a) If the new residuals  $y_i$  are greater than the previous ones, set

$$B_{ij} = B_{ij} + \lambda_{ii}$$

Solve the system again and continue solving until the new residuals are less than the old. Now the system is ready for a new iteration.

b) If the new residuals  $y_i$  are less than the old, set

$$B_{ij} = B_{ij} - \lambda_{ii}$$

Solve the system again and continue solving until the new residuals are greater or equal to the old.

The iteration continues until either the maximum number of iterations (input) is exceeded or the residuals are less than or equal to an input tolerance.

## S. EQUATIONS FOR FLIGHT PATH AZIMUTH AND FLIGHT PATH ANGLE

A subroutine computes the flight path azimuth and flight path angle with the following equations:

### 1. Flight path angle

$$\gamma = \sin^{-1} \left[ \frac{\dot{R}}{V} \cdot \hat{N} \right] \quad (S.1)$$

$\hat{N}$  is the vertical unit vector. In the geodetic system  $\hat{N}$  is given by

$$\hat{N} = [\cos \phi \cos (\theta - \theta_0), \cos \phi \sin (\theta - \theta_0), \sin \phi]$$

In the geocentric system  $\phi$  is replaced by  $\phi'$ . Alternatively, in the latter system

$$\hat{N} = \frac{R}{r}$$

### 2. Flight path azimuth

$$A = \sin^{-1} \left[ \frac{1}{\cos \gamma} \left\{ \frac{\dot{y}}{V} \cos (\theta - \theta_0) - \frac{\dot{x}}{V} \sin (\theta - \theta_0) \right\} \right]$$

$$A = \cos^{-1} \left[ \frac{1}{\cos \gamma \cos \phi} \left\{ \frac{\dot{z}}{V} - \sin \gamma \sin \phi \right\} \right] \quad (S.2)$$

Both formulas are used to determine the proper quadrant of  $A$ . To obtain the geocentric output,  $e^2 = 0$ ,  $\phi$  is replaced by declination  $\delta = \phi'$ .

## T. OSCULATING ELEMENTS

The osculating elements are obtained from the following equations:

$$a = \left( \frac{2}{r} - \frac{v^2}{\mu} \right)^{-1} \quad (T.1)$$

$$n = \mu^{1/2} |a|^{-3/2} \quad (T.2)$$

$$\left. \begin{array}{l} e \cos E \\ e \cosh E \end{array} \right\} = 1 - \frac{r}{a} \quad (T.3)$$

$$\left. \begin{array}{l} e \sin E \\ e \sinh E \end{array} \right\} = \frac{d}{\sqrt{|\mu a|}} \quad (T.4)$$

$$M = \left\{ \begin{array}{l} E - e \sin E \\ e \sinh E - E \end{array} \right. \quad (T.5)$$

$$t_p = t - \frac{M}{n} \quad (T.6)$$

The angles  $\Omega$ ,  $\omega$ ,  $i$  are obtained from the vectors  $H$  and  $\hat{P}$ , where

$$H = R \times \dot{R} \quad (T.7)$$

$$eP = \left( \frac{1}{r} - \frac{1}{a} \right) R - \frac{d}{\mu} \dot{R} \quad (T.8)$$

In terms of these vectors:

$$\cos i = \frac{H_z}{h} \quad \text{in the first or fourth quadrant} \quad (T.9)$$

$$\sin \Omega = \frac{H_x}{h \sin i} \quad (T.10)$$

$$\cos \Omega = \frac{-H_y}{h \sin i}$$

$$\cos \omega = P_x \cos \Omega + P_y \sin \Omega \quad (T.11)$$

$$\sin \omega = \frac{P_z}{\sin i}$$

## U. IMPACT PARAMETERS

The "impact parameters" are coordinates in the "impact" plane. This plane passes thru the body (T-planet or the moon) and is normal to the incoming asymptote. The direction cosines of the asymptote are given by equations (U.1, U.2) in terms of unit vectors  $\hat{P}$  (Appendix T) and

$$\hat{Q} = \frac{H}{h} \times \hat{P} \quad (U.1)$$

$$\hat{S} = \frac{1}{e} \left( \hat{P} + \sqrt{e^2 - 1} \hat{Q} \right) \quad (U.2)$$

In the plane defined by  $\hat{S}$  as its normal, two unit vectors  $\hat{T}_{IMP}$  and  $\hat{R}_{IMP}$  are defined.  $\hat{T}_{IMP}$  is parallel to the ecliptic plane for mars and venus impacts and to the moon's orbital plane for moon impacts. Explicitly

$$\hat{T}_{IMP} = \frac{\hat{N} \times \hat{S}}{|\hat{N} \times \hat{S}|} \quad (U.3)$$

where  $\hat{N}$  is the unit normal to the ecliptic plane, or the moon's orbital plane.  $\hat{R}_{IMP}$  is normal to both  $\hat{S}$  and  $\hat{T}_{IMP}$ .  $\hat{B}_{IMP}$  is the vector from the body to the vehicle as it crosses the impact plane. The information computed are the dot products

$$B_{IMP} \cdot \hat{T}_{IMP} \text{ and } B_{IMP} \cdot \hat{R}_{IMP}$$

## W. EQUATIONS FOR TRANSLUNAR PLANE INPUT

The translunar plane input is designed to permit easy visualization of the geometric relationships between initial conditions for circumlunar trajectories and the motion of the moon. (See Figure 1).

The initial conditions are given in a coordinate system referred to the translunar plane. This system has its x-axis along the ascending node of the vehicle with respect to the moon's orbital plane, its y-axis in the translunar plane, at right angles to the ascending node, in the direction of motion. In this coordinate system, initial position and velocity vectors are given by

$$\begin{aligned} x_{TL} &= (r_B + h) \cos \Psi \\ y_{TL} &= (r_B + h) \sin \Psi \\ z_{TL} &= 0 \end{aligned} \quad (W.1)$$

Here  $r_B$  is the radius of the body of departure (earth or moon).

$$\begin{aligned} \dot{x}_{TL} &= v \sin (\gamma - \Psi) \\ \dot{y}_{TL} &= v \cos (\gamma - \Psi) \\ \dot{z}_{TL} &= 0 \end{aligned} \quad (W.2)$$

The translunar plane is positioned by giving its inclination  $i_{TL}$  with respect to the moon's orbital plane and the lunar lead angle  $\phi$ , the angle between the moon's position at injection and the descending node. The vectors  $R_{TL}$  and  $\dot{R}_{TL}$  may then be transformed into the equatorial system by the following series of rotations:

1. A rotation  $-i_{TL}$  about the  $x_{TL}$  axis will rotate the translunar plane into the moon's orbital plane.

2. A rotation of  $\pi - (\lambda_M + \phi)$  about the new z-axis will refer the moon's orbital plane coordinate system to the ascending node of the moon's orbital plane (with respect to the equator) as x-axis.

Here  $\lambda_M$  stands for the argument of latitude of the moon. These rotations are performed by multiplying  $R_{TL}$  and  $\dot{R}_{TL}$  by the matrix:

$$A = \begin{pmatrix} -\cos(\lambda_M + \phi) & \sin(\lambda_M + \phi) & -\sin(\lambda_M + \phi) \sin i_{TL} \\ -\sin(\lambda_M + \phi) & -\cos(\lambda_M + \phi) \cos i_{TL} & \cos(\lambda_M + \phi) \sin i_{TL} \\ 0 & \sin i_{TL} & \cos i_{TL} \end{pmatrix} \quad (W-3)$$

3. The moon's orbital plane (MOP) is rotated about its node through an angle  $-i_M$  (the inclination of the MOP).

4. The ascending node is brought into coincidence with the vernal equinox by a rotation  $-\Omega_M$ . These two rotations are embodied in the matrix

$$B = \begin{pmatrix} \cos \Omega_M & -\sin \Omega_M \cos i_M & \sin \Omega_M \sin i_M \\ \sin \Omega_M & \cos \Omega_M \cos i_M & -\cos \Omega_M \sin i_M \\ 0 & \sin i_M & \cos i_M \end{pmatrix} \quad (W-4)$$

and thus:

$$R = (BA) R_{TL} \quad (W.5)$$

$$\dot{R} = (BA) \dot{R}_{TL}$$

Y. CHANGE OF INDEPENDENT VARIABLE - BETA MODE

According to the standard Encke method, we introduce a differential equation

$$\ddot{\rho} = -\mu \frac{\rho}{|\rho|^3} \quad (Y. 1)$$

In the construction of the closed-form solution for (Y. 1), a parameter  $\beta$  arises. It is related to  $t$  by Kepler's equation,

$$t = t_0 + \frac{f(\beta)}{\sqrt{\mu}} \quad (Y. 2)$$

where  $f$  is a transcendental function of  $\beta$  and is obtained by summing several power series.

If  $t$  is taken as the independent variable, Equation (Y. 2) has to be solved for  $\beta$  by an iterative method, requiring numerous time-consuming evaluations of the function  $f$  for each integration step. Using  $\beta$  as the independent variable, however, only requires a single evaluation.

It remains, of course, to see what becomes of Equation (Y. 1) and

$$\begin{aligned} \ddot{\xi} &= \ddot{x} - \ddot{\rho} \\ &= -\mu \left( \frac{x}{|x|^3} - \frac{\rho}{|\rho|^3} \right) + F \end{aligned} \quad (Y. 3)$$

if  $\beta$  is the independent variable. We have, from Kepler's equation, that

$$\frac{dt}{d\beta} = \frac{|\rho|}{\sqrt{\mu}} \quad (Y.4)$$

at any point along the solution of (Y.1). Thus

$$\dot{\rho} = \rho' \frac{\sqrt{\mu}}{|\rho|} \quad \text{and} \quad \rho' = \dot{\rho} \frac{|\rho|}{\sqrt{\mu}}$$

at any point along the solution of (Y.1) and the initial conditions become

$$\rho(\beta_0) = x_0 \quad \text{and} \quad \rho'(\beta_0) = \frac{\dot{x}_0 |\mathbf{x}_0|}{\sqrt{\mu}}$$

when

$$\beta_0 = \beta(t_0) = 0$$

Now the solution for (Y.1),  $\rho$  and  $\rho'$ , can be written in closed form for any  $\beta$ . As auxiliary quantities in this solution, we have  $|\rho|$  and  $D = \frac{\rho \cdot \rho'}{|\rho|}$ . They are computed as functions of  $\beta$  before  $\rho$  and  $\rho'$  are known; that is, with accuracy at least as good as that of  $\rho$  and  $\rho'$ . Not only are they needed and easy to compute, but they also have the interesting property that

$$\frac{dt}{d\beta} = \frac{|\rho|}{\sqrt{\mu}} \quad (Y.5)$$

and

$$\frac{d^2t}{d\beta^2} = \frac{D}{\sqrt{\mu}}$$

Thus Equation (Y.1) is solved more economically in terms of  $\beta$  than in terms of  $t$ .

- Now we turn to Equation (Y.3). To treat it, we want to express  $\ddot{\xi}''$  in terms of  $\ddot{\xi}$ . From (Y.5) we have that

$$\ddot{\xi}' = \ddot{\xi} \frac{dt}{d\beta} = \ddot{\xi} \frac{|\rho|}{\sqrt{\mu}} \quad (Y.6)$$

Differentiating with respect to  $\beta$ ,

$$\begin{aligned} \ddot{\xi}'' &= \frac{d\ddot{\xi}}{d\beta} \frac{|\rho|}{\sqrt{\mu}} + \ddot{\xi} \frac{d}{d\beta} \left( \frac{|\rho|}{\sqrt{\mu}} \right) \\ &= \ddot{\xi} \left( \frac{|\rho|}{\sqrt{\mu}} \right)^2 + \ddot{\xi}' \frac{\sqrt{\mu}}{|\rho|} \frac{D}{\sqrt{\mu}} \\ &= \ddot{\xi} \frac{|\rho|^2}{\mu} + \ddot{\xi}' \frac{D}{|\rho|} \\ &= -|\rho|^2 \left( \frac{x}{|x|^3} - \frac{\rho}{|\rho|^3} \right) + \frac{|\rho|^2}{\mu} F + \ddot{\xi}' \frac{D}{|\rho|} \end{aligned} \quad (Y.7)$$

Thus (Y.7) is the equation to be integrated numerically, instead of

(Y.3). The coefficients  $\frac{|\rho|^2}{\mu}$  and  $\frac{D}{|\rho|}$  can be calculated with much more accuracy than the factors involving  $\ddot{\xi}$ , since they depend only on the two-body solution. For analysis of error propagation, we write (Y.7) as

$$\ddot{\xi}'' = -\frac{1}{|\rho|} \left[ (\rho + \xi) \frac{|\rho|^3}{|\rho + \xi|^3} - \rho \right] + \frac{|\rho|^2}{\mu} F + \ddot{\xi}' \frac{D}{|\rho|} \quad (Y.8)$$

The mechanics of the procedure, then, are easy to enumerate. The initial conditions are  $x_0$  and  $\dot{x}_0$ . Let

$$\begin{aligned}\rho(t_0) &= x_0 \\ \rho'(t_0) &= \frac{\dot{x}_0 |x_0|}{\sqrt{\mu}}\end{aligned}\tag{Y. 9}$$

Using these initial conditions, evaluate  $t$ ,  $\frac{|\rho|^2}{\mu}$ ,  $\frac{D}{|\rho|}$ ,  $\rho$ ,  $\rho'$  for each value of  $\beta$  to be considered.

Let  $\xi_0 = \xi'_0 = 0$ . Using these initial conditions, integrate Equation (Y. 7) to get  $\xi(\beta)$  and  $\xi'(\beta)$ . Note that the first two terms on the right-hand side of Equation (Y. 7) are functions of  $x$  and possibly  $x'$ . These are obtained by

$$\begin{aligned}x(\beta) &= \rho(\beta) + \xi(\beta) \\ x'(\beta) &= \rho'(\beta) + \xi'(\beta)\end{aligned}\tag{Y. 10}$$

If at any point  $\dot{x}$  is required, it can be found from

$$\dot{x}[t(\beta)] = x'(\beta) \frac{\sqrt{\mu}}{|\rho(\beta)|}\tag{Y. 11}$$

Depending on the rectification control logic, there will be places where the solution to Equation (Y. 1) must be started over. At this point, the values  $t$ ,  $x$ ,  $\dot{x}$  become the new  $t_0$ ,  $x_0$ , and  $\dot{x}_0$ , while  $\beta$ ,  $\xi$ , and  $\xi'$  are reset to zero.

### Comparison of Modes

- a) It is immediately apparent that eliminating the necessity of iteratively solving Equation (Y. 2) will substantially increase the speed of computation.
- b) An important advantage arises further from eliminating the sometimes ponderous logic which supplies initial guesses for the iterative process and guarantees convergence of the solution.
- c) A third advantage of the  $\beta$ -method is not quite so apparent, but no less important. It is well known that the size of the integration time step can be increased as the distance from the center of attraction increases. This change of the integration interval requires a cumbersome restart procedure. An examination of Equation (Y. 5) shows that equal intervals of  $\beta$  correspond to time intervals of increasing lengths as the distance increases. The time interval thus automatically expands and contracts correctly without outside intervention.
- d) Geometric stopping and printing conditions can usually be conveniently expressed in terms of  $\beta$ , whereas they often require iterative determinations of the time. This advantage, however, is slight.

e) If state vectors are required at fixed times, an iteration is necessary to find the corresponding value of  $\beta$ . In this case the  $\beta$ -method is no better, and no worse, than the standard methods. If such vectors are required at frequent, closely spaced, time points (as in orbit determination, for instance), the advantage of the  $\beta$ -method is marginal.